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ABSTRACT

This volume contains the proceedings of the International Conference on Technology in Mathematics Education (ICTME) held at the Lebanese American University in July, 2000. The conference included a panel discussion on technology and the new curriculum, hands-on workshops, and focus group discussion centered around the themes of the effects of technology on changing the role of teachers, changing the role of students, and changing assessment. Papers include: (1) "Rethinking Mathematical Learning with Digital Technologies" (Celia Hoyles and Richard Noss); (2) "Educating Students for Their Future Not Our Past: A Challenge for Teachers of Mathematics" (Peter Jones); (3) "Technology and Problem Solving in Mathematics: Myths and Reality" (Murad Jurdak); (4) "Learning with Multimedia--Mathematics Needs a Special Approach" (Bernard Winkelman); (5) "Calculus at the Start of the New Millennium" (Deborah Hughes Hallett); (6) "Distance Learning between German and Japanese School Classes Based on a Real Time Video Conference Environment" (Klaus-D. Graf); (7) "Attitudes and Concerns on Distance Learning in Lebanon: A Multiple-Case Study" (Ramzi Nasser and Kamal AbouChedid); (8) "A Teacher's Experience in Developing a Set of Interactive Computerized Tests" (Nada Alamedine); (9) "Some Current Developments in the Production and Application of Interactive Mathematics Teaching/Learning Modules at the Higher Colleges of Technology in the UAE" (Leonard Raj and Khaled Abdullah); (10) "Using Technology as a Tool for Teaching Mathematics at the Secondary School" (Mary Nabbout and Bilal Basha); (11) "The Multimedia in Our Mathematics Classroom" (Mohamad Mounir Fakhri); (12) "Non Trivial Applications of MAPLE in Teaching Mathematics" (Miroslaw Majewski); (13) "Developing Internet Resources for Online Teaching Mathematics with Scientific Notebook" (Miroslaw Majewski); (14) "Euler-type Formula Using Maple" (Badih Ghusayni); (15) "WWW Mathematics at the University of Pretoria" (Johann Egelbrecht and Ansie Harding); (16) "An Alternative Sequence for the Calculus" (Kamel Haddad); (17) "An Anti-Essentialist View about ICT in Mathematics Education: What Differences Can It Make to Mathematics Teacher Education" (Bibi Lins); (18) "Is It Just a Computer?" (Christine Sabieh); (19) "Mathematical Technology: In the Hand or On the Desktop" (N.V. Challis and H.W. Gretton); (20) "Multi-Objective Optimization in Computer Aided Control System Design" (Hussein Sayed Tantawy); (21) "The Effect of Using the Geometer's Sketchpad (GSP) on Jordanian Students' Understanding of Geometrical Concepts" (Farouq Almeqdadi); (22) "Jordan Experience with Computer Based Instruction in

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International Conference on Technology in Mathematics Education

July 5 - 7, 2000
Beirut, Lebanon

LEBANESE AMERICAN UNIVERSITY

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INTERNATIONAL CONFERENCE ON TECHNOLOGY IN MATHEMATICS EDUCATION

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ACKNOWLEDGEMENTS

The organizers of ICTME 2000 wish to thank the volunteers who worked long hours to help create this event, and the Lebanese American University, administration and staff for the support it has given in organizing this event. We wish to thank also the contributions of our student assistants, in particular Samer Younes, webmaster and main compiler of the papers for the proceedings.

The conference also acknowledges the following organizations for their generous support:

UNESCO (Beirut Office)

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A Note from the Local Committee

In 1992 Lebanon emerged from a civil war that had lasted for 15 years, destroying its physical as well as its human infrastructure. Since then, the country at all levels has turned into a huge reconstruction site: While the government is building the physical infrastructure and is trying to improve the level of education in its schools and universities, private institutions embarked on projects to catch up with the developments in all fields and specialties. One can say that Lebanon is quickly regaining its position as the educational center of the Middle East.

At the Lebanese American University, administrators and faculty members have been very active trying to upgrade the level of the institution. Mathematics faculty members, in particular, have reviewed the math curriculum, made some changes in its content and adopted new teaching practices. Computers are frequently, but appropriately used in most classes, and visualization has become an essential part of the learning process. On the national level, a workshop on using advanced calculators in the teaching of mathematics was organized in the summer of 1998. The workshop aimed at high school instructors and was given by Adrian Oldknow, the co-chair of this conference. In the following year, a follow-up one-day conference was organized, which included 75 mathematics teachers from various schools in Lebanon and in the summer of 2000 our efforts culminated in the International Conference on Technology in Mathematics Education (ICTME). The year 2000 was very special to Lebanon and LAU: the university was celebrating its 75th anniversary, and the country was celebrating the end of 22 years of ruthless Israeli occupation of Southern Lebanon.

ICTME aimed at serving as a forum for an in-depth treatment and informal exchange of ideas to promote the creative use of technology in Lebanon and the neighboring Arab countries. By inviting renowned specialists in the field from the Western world, the conference provided means for a dialogue between practitioners and researchers in developed and developing countries. University and high school teachers from Lebanon, Jordan, Egypt, The United Arab Emirates, the UK, Germany, South Africa, and Macau contributed papers providing a rich and varied scope of experiences in technology based education.

The program of the conference also included a panel discussion on Technology and the New Curriculum. This was chaired by Samer Habre and it included Nimr Freiha, director of the Educational Center for Research and Development, May Abboud, Iman Osta and Adrian Oldknow. Furthermore, the following hands-on workshops were held: *Mathematica and Calculus – A Tutorial* run by May Abboud and Samer Habre; *Hand-Held Calculators and High School Mathematics* run by Adrian Oldknow and Neill Challis; *An Introduction to Maple* run by Elias Deeba. In addition, the conference featured focus groups that discussed the following themes:

- How does technology change the role of the teacher? Does it diminish it, enhance it, or simply modify it?
- How is the role of the learner modified when technology is used in the teaching of Mathematics? Does this imply a restructuring of the classroom?
- How is assessment affected by the introduction of technology in the teaching of Mathematics?

Reports on the outcomes of the focus groups meetings were presented at the end of the conference.

Plans are under way for the second ICTME (summer 2002). In the mean time, we would like to thank all the members of the International Committee for their support, the invited speakers for accepting our invitation, the participants for believing in us, the university administrators and the sponsors for making this event possible.

Foreword

I am delighted to take this opportunity to congratulate the local organizing committee at LAU for their initiative, hard work and skill in making this conference such a success. On behalf of those coming to Beirut from abroad I would like to thank all those local participants, contributors and organizers who made us so welcome, and showed us just how wonderful Lebanese hospitality is. I very much hope that this conference will pave the way for future international contacts in the region on the use of ICT to enhance the teaching of mathematics.

Mathematics and ICT have an interestingly symbiotic relationship. Computers, and other ICT products, rely heavily on a range of smart mathematical ideas which have been developed largely over the past 50 years or so. My own major interest is in computer graphics - without which we would not have had algorithms for the accurate drawings on old-fashioned XY-plotters which were needed to produce the "masks" required for the production of the original microprocessors. But there are many other branches of mathematics, which have made significant contributions - perhaps one of the most well documented one being the contribution of number theory to the secure transmission of information. On the other hand computers are now taken for granted as research tools by mathematicians in almost all branches of the subject, and many new results have been obtain with computer aid, which would otherwise have been unfeasible. This includes my own discoveries and proofs in the geometry of the triangle.

Yet we have a much more ambivalent approach to the use of computers in teaching mathematics. Sometimes there is the feeling that we insist on students doing some things "the old-fashioned" way, more because we had to endure it, than because it makes sense! In any case it is quite possible to develop superb computer-assisted learning tools to help teach aspects of mathematics which themselves have become quite obsolete! The existence of powerful ICT tools for teaching does not by itself imply any re-examination of the sets of skills, knowledge and understanding which should underpin the mathematical education for a technological era. This requires educators with the foresight, judgment and commitment to address the task, often in the face of considerable opposition from their own colleagues. As a contribution to those of you engaged in such a task I offer a little model of my own which I have used from time to time to suggest to those of my own skeptical colleagues why we need to engage with the task. I call it the **STAR** model, and its aim is to widen the focus of mathematical activity to consider a range of skills we now expect from those involved in mathematical problem-solving at a wide range of levels (including research). My suggestion, then, is that in tackling mathematical problems we assume a variety of quite different roles as:

- **Strategist**: selecting from a range of possible techniques and representations ones which we judge, mainly from experience, will be most likely to result in a successful approach;

- **Technician:** performing a range of calculations, manipulations and simplifications using the chosen techniques and representations towards an ultimate solution;

- **Accountant:** making careful checks that we haven't made mistakes, that our assumptions are justified, that our solutions are realistic..

- **Reporter:** communicating our results to fellow mathematicians and interested third parties through e.g. seminars, lectures, conference contributions, published articles, chapters in books…

It is interesting to see the extent to which ICT can support, or even displace, the human in each stage of this process. Some highly respected jobs - such as navigators for airlines, and draughtsmen for engineering companies - have been wiped out through the power of computers to carry out the routine processing tasks associated with those posts. We need to take a broad and realistic look at why we think we are educating our students the way we currently do to ensure that we are not in the process of just turning out obsolescent technicians! It is for this reason that I believe such conferences as last July's ICTME conference in Beirut have such a vital part to play in the exchanging of views, ideas, experiences and approaches to the benefit of all participants.

Adrian Oldknow, co-chair of ICTME

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INVITED SPEAKERS

Rethinking Mathematical Learning with Digital Technologies[1]

Celia Hoyles & Richard Noss
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University of London

ABSTRACT:

In this paper, we reflect on the ways in which newly-created alternatives to textual forms of representation are redefining the utility and power of computational environments — microworlds — and offering advantages (as well as disadvantages) for mathematical learning in the sense of understanding how things work and why. We will compare two different forms of computational environments we have built which are separated by half a decade — an important half-decade in the evolution of what is possible with affordable computers.

Some time ago, we built a microworld, Mathsticks, (written in Microworlds Logo) which involves the learner in constructing sequences of objects on the screen, and manipulating them directly (for details, see Noss, Hoyles & Healy, 1997). The key interesting feature of this world is that while the user is manipulating objects *directly*, there is also graphical feedback as to the results of their actions and a visible (conventional, Logo) program constructed to provide a formal representation of what has been done. By exploiting the relative advantages of direct manipulation and text-based programming, we aimed to construct an environment in which users would learn by linking their actions with the graphical or the symbolic representations — both representations were consistently available. We have noted in the past the problems that children have with maintaining a connection in Logo programming between the symbolic commands and their visual effects (Hoyles and Sutherland, 1989). In terms of elementary geometry, Turtle Math has gone a long way to overcome this problem through its maintenance of a close correspondence between representations and the bidirectionality between the different modes — that is the student can move the mouse and the Logo commands are created automatically (see Clements & Sarama, 1995, Clements, Battista, Sarama, Swaminathan, 1996). We tried to build something similar in our Mathsticks microworld.
The main feature of the microworld was that purpose-built tools were designed to simulate the arrangement of matches in a sequence (see Figure 1), with the mathematical goal that students would come to appreciate number patterns as functional relationships.
Here we simply summarise our agenda and findings. First the students would be asked to build by direct manipulation of the screen matches, a model of how they saw a given term in a sequence of matches. For example in Figure 1, the students had constructed the fifth term of a sequence of squares. Through this process of construction, we anticipated that students would come to notice the structure of the given term in itself and in relation to others; they would begin to see the particular case as an instance of the general. Our expectation stemmed from two sources: firstly because their mental image of the task could be reflected in their actions in the microworld (for example picking up and putting together the matchstick icons shown on the top right of the screen), and in the rhythm of their pointing and clicking; and

[1] Part of this paper is a modification of a paper submitted to the Journal of Educational Computing

secondly because it could be realisea simultaneously in symbols (programming code) appearing in the box labelled 'history' and visually as an array of graphical objects.

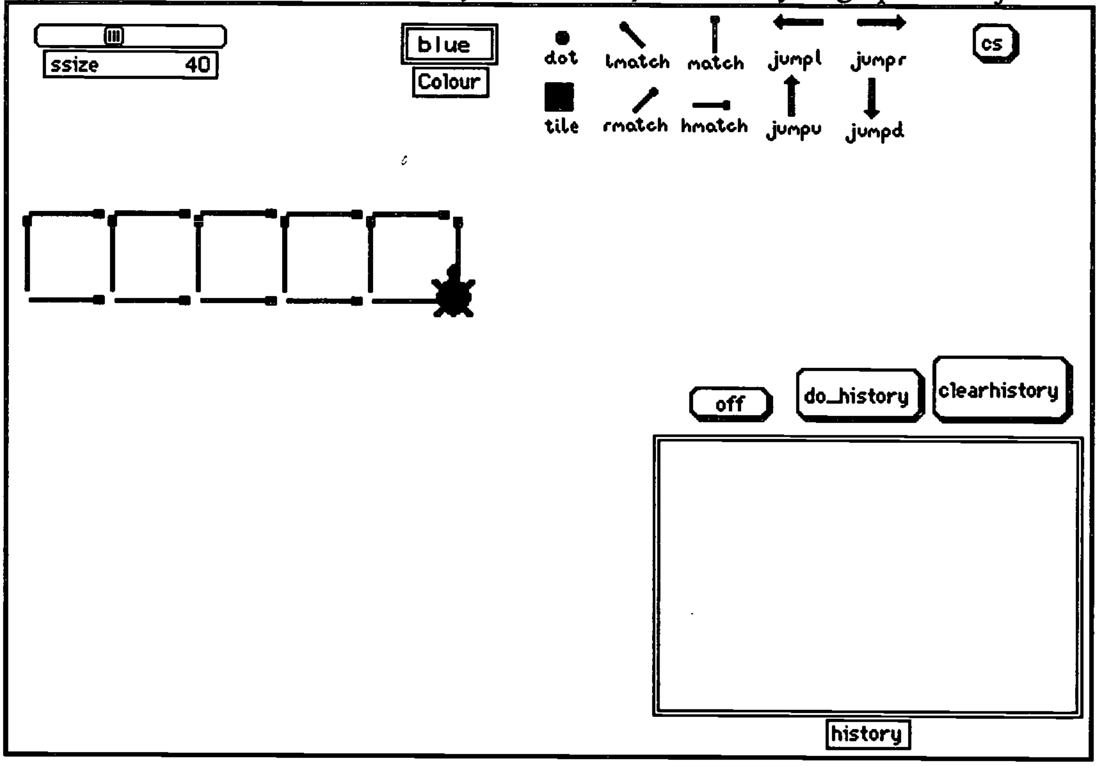


Figure 1:An example screen from the Mathsticks microworld

(For details of how the different images the students held of the task, see Healy and Hoyles, 1999).

But Logo did not only serve the role of a symbolic representation of the mathematics involved. We encouraged the students to write a general Logo procedure that could be used to build any term of the sequence. This, we hoped, would force the distinction between those aspects of their code that were significant for sequence structure from those that represented the characteristics of a particular term. In this way too, the dynamic algebra of the programming code would — and did — become a means for them to think about the relationships in the sequence.

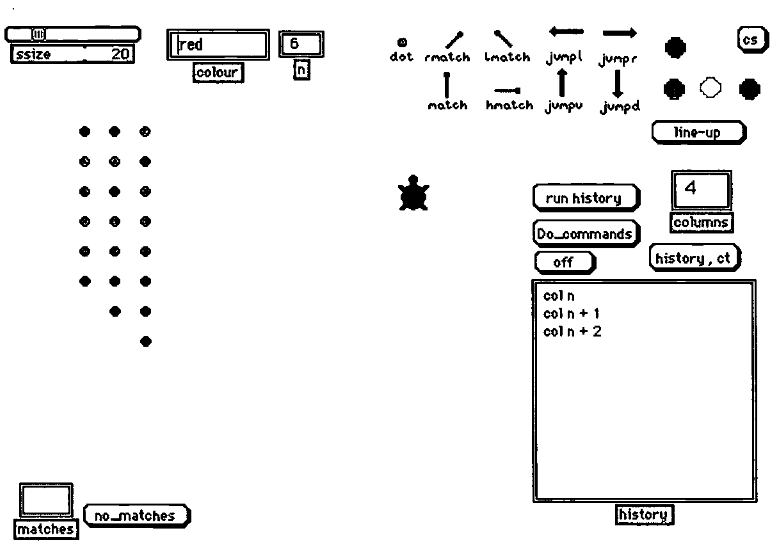
From Constructing to explaining:

We used this software as part of a set of activities aiming to help students move from informal argumentation to more formal explanation and proof in mathematics. Over the last few years, Lulu Healy[2] and Celia Hoyles have devised algebra and geometry teaching sequences with this aim in mind. Our activities were developed after analysing students' responses to a nationwide paper-and-pencil survey to assess students' conceptions of proving and proof (see Healy and Hoyles, 2000). This questionnaire was completed by 2,459 fifteen year-old students of above average mathematical attainment from across England and Wales. This showed that most students knew that a valid proof should be general and valued arguments that explain.

[2]ESRC project, *Justifying and Proving in School Mathematics,* Ref R000236178. I wish to acknowledge the central work of Lulu Healy in all aspects of this project.

Many students could also present relevant information for a proof (informal explanation or empirical observations), but rather few were able to make the transition to algebraic proof or to deductive proof in geometry.
Activities were designed (see also see Hoyles and Healy 1999) where, through computer construction, students have to attend to mathematical relationships and in so doing are provided with a rationale for their necessity. Thus, students were to construct mathematical objects for themselves on the computer, conjecture about the relationships between them, and check the truth of their conjectures with the tools available. The teaching sequence also was to include reflection away from the computer guided by the teacher, and the introduction of mathematical proof as a particular way of expressing one's convictions and communicating them to others. It is in this way, we hoped that constructing and proving could be brought together in ways simply not possible without an appropriate technology.
One of the activities involved an investigation of the properties of the sums of different numbers of consecutive numbers.

The Mathsticks environment was extended to included dots as a well as matches. Numbers could be represented as columns of dots as shown in Figure 3



The dots could be planted directly or by writing Logo code. After some experimentation students convinced themselves that the sum of three consecutive numbers was always divisible by 3.

Figure 5 presents an attempt to prove that the sum of five consecutive numbers is always a multiple of five.

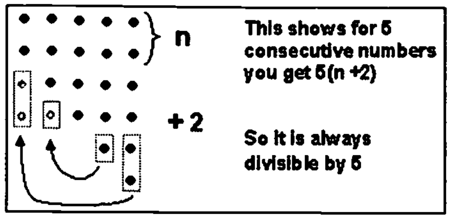
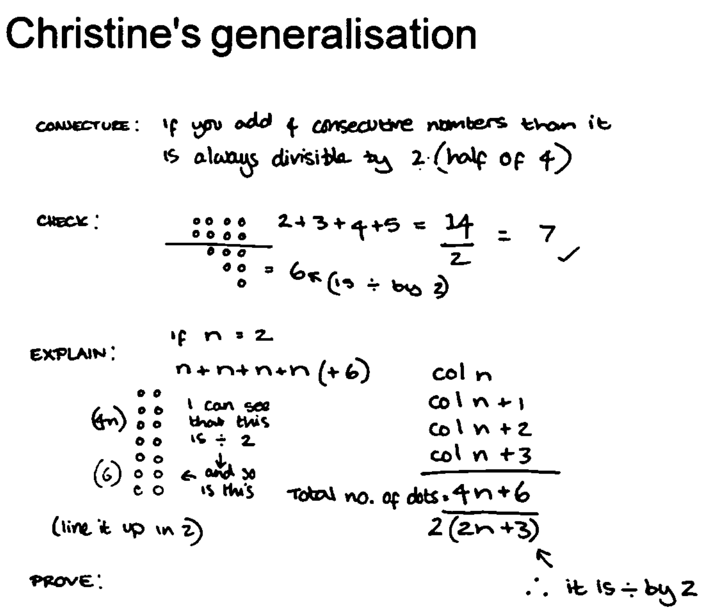


Figure 5: Manipulating to prove

It was written by a student who first constructed a variable Logo procedure to generate a column of n dots, and then used this to produce a visual representation of the five consecutive numbers 2, 3, 4, 5 and 6. The student manipulated the figure in such a way that the conjecture and its proof emerged simultaneously – in one moment the student identified both *that* and *why* the property holds. No more examples were deemed to be necessary as there was nothing special about the choice of 2 for the first number – or rather what was special about it was that it represented both the variable n and the first 2 dots in every column.

They then went on to make informal conjectures about the sum of four consecutive numbers: for example they suggested that it was divisible by 2 but not divisible by 4. They then tested and verified their conjectures using the computers tools and then set out to explain and prove hem in algebra. Figure 4 shows the proof written by one of our students. It shows how the software scaffolded her algebraic approach and helped here to write a general and formal proof



Beyond Mathsticks:

Our findings convinced us of the power of the Mathsticks microworld and its potential for learning. Our conclusion was that it was the interplay between learners' actions and the different representations of the mathematical relationships embedded in the microworld that was crucial to our students' learning. It seemed to us that we were not only seeing the importance of students building connections between multiple representations — although this is of course of considerable importance (see for example software such as SimCalc; Kaput and Roschelle, 1999). The design of Mathsticks aimed for more: we were strongly influenced by the notion of a general programming language from which to build an application, and we saw this as generative of expressive power, beyond that which is possible with the multiple representation approach. It is instructive to consider this approach in the light of the 'programmable application' proposed by Eisenberg, 1994, although this starts at the applications end. His ideal is, however, one with which we have considerable sympathy: to create software that 'allows students to work with both a powerful (and ideally learnable) interface and a powerful (and ideally learnable) programming language (*ibid.*, p 181). It is this gain in expressive power which is also at the heart of diSessa's Boxer idea: in this case, the interface and the language are so tightly integrated that it is sometimes difficult to know whether one is programming are 'just' editing text.

We want to draw attention to two facets of Mathsticks that we regard as important. In the first place, *we provided tools to focus on what mattered to us* — just enough of the relevant structures were embedded in the tools so that they could be thought and talked about, explored and manipulated, at a level 'above' that of Logo. There is an important methodological point here. In order to study the forms of expression employed by learners to mediate their understandings of mathematical ideas, we must tread a careful path between allowing free range to that expression and constraining it too tightly. In the former case, it might be difficult to capture any traces of the learner's thinking — let alone any traces that could be used as resources for mathematical learning. The latter approach runs the risk of constraining thinking to the point of total predictability, in which case we would merely be studying our own preferences (this phenomenon is closely connected with the idea of the didactic contract, Brousseau, 1984, although here we are concerned with constraints built into tools while Brousseau's construct concerns teachers' constraining behaviour). As researchers, let alone educators, we have to respect *and* constrain diversity.

The second point is that at *the same time, nothing was ruled out*: the students had a working knowledge of Logo, knew how it worked in the sense of editing and writing programs, changing interface objects, and were free to adopt any method of construction. Thus the tools could be manipulated by programming, as well as by direct manipulation in a way that maintained connections between the Mathsticks interface level and the level of the language below.

In trying to make deeper sense of the microworld idea, we will now attempt to clarify some of the casual references we have made so far about levels, and introduce the distinction between *platform* and *superstructure*. By platform, we mean the base level at which it is possible for users (rather than professional programmers) to interact. A platform would include high level programming languages but not for example machine code. In most cases, users interact with the platform because the designer *expects* them to do so. Most software (including educational software and, as we mentioned earlier, even some applications in Logo) takes pains to make the platform level *completely* invisible, and, in general, make a virtue out of this perceived necessity on the grounds that only programmers need to know how to program.

Superstructure, on the other hand, describes the objects in the microworld and ways to manipulate them (the matches in Mathsticks). For the moment, it doesn't matter whether this form of manipulation is direct (e.g. via a mouse or speech input), or by entering lines of text which may or may not be the same as the language of the platform. The point is that the kinds of interactions which users' experience and the

HCI tools they employ are a subject for the designer, and determine to a great extent what activities and experiences the user has as she interacts with the program. For most users with most software, there is only superstructure.

The idea of superstructure raises new dilemmas. How visible is the platform level? How easy is it for the user to 'descend' to the platform level? How permeable is the barrier that separates superstructure from platform? How rich is the potentiality of modifying tools at the superstructural level along with the interactions that go with it? How familiar can and should the user be with platform tools?

The answer, of course, depends on the pedagogical aim. The issue is not, of course, whether the mechanisms should be open to the user, but *how much* of them should be. But there is a problem, and this time it *is* primarily technical. Whereas it makes sense to maintain a certain eclecticism regarding user interface at the superstructural level (as we did with Mathsticks), it is very difficult to adopt the same approach as regards the platform, where text is all there is.

We should not underestimate this distinction — non-linguistic (iconic, GUI) interfaces are now *the* way that people expect to interact with machines. They do not, in general, expect to program in any conventional sense. There is therefore an interaction barrier[3]: the things you have to do to gain a sense of the microworld's mechanism may be substantially different from the things you do within the microworld. It is not just the things on the screen that are different; it is the ways you move them around that conventionally distinguishes platform from superstructural interaction. *Finding ways to break down this distinction may turn out to represent a significant advance for mathematical learning with digital technologies.*

This issue raises a second major question of microworld design, which is related to but separate from the interface issue above. What is the appropriate grain size for objects and relationships at the superstructural level? What level of complexity is appropriate for users; how far, in other words, should be the distance between superstructural elements and the platform on which they are built? We cannot hope to answer this question in the abstract; it is a research question which we (and many others) are currently addressing, and which we will discuss below in the context of our current work.

The Playground project: aims and methodologies:

In our ongoing research project we have moved to a new arena for constructionism — that of computer games. In this, we are building on the work of Harel and Kafai (Harel, 1988 and Kafai, 1995) who have pointed out that we should endeavour to tap in to children's games culture by adding a new dimension whereby they build their *own* games. Our project is to design and try out computational worlds — playgrounds — in which the objects in a game *and* the means for expressing them are engaging; where the programming of a game is itself a game. We have set ourselves the task of working with young children (aged as young as 4 and at most 8) where it is obvious that we cannot rely on the written word as a means of communication. This has challenged us considerably and forced us to take seriously other modalities of interaction, such as speech as well as direct manipulation. We want to give the

[3] We use this term by analogy with the idea of *abstraction barrier* in computer science, the means by which programmers can wrap up pieces of programs to use as tools whose workings need not be visible in the larger program. This is a key means of controlling complexity: 'By isolating the underlying representations of data objects, we can divide the task of designing a large program into smaller tasks that can be performed separately" (Abelson and Sussman, 1985, p.126).

children the opportunity to construct creative and fun games, and at the same time, offer them an appreciation of — and a language for — the rules, which underpin them.

In real playgrounds, children play with the objects they find, like swings and roundabouts, balls and sticks. They create new games with new rules (stand on the swings, have three people on the seesaw), make up new games with the existing objects (who can go fastest around the seesaw, climbing frame and slide?), and ignore the objects they find to create new ones (like hopscotch). And in a real playground, kids engage in a wide range of personal styles — through talking and shouting, moving, hearing seeing and so on. All their senses are engaged.

Our virtual playgrounds aim at a similar level of engagement. Unlike most computer games, in which the metaphors are simple, and the tools immediate to provide entertainment, our games are being constructed for learning. And to achieve this, we have had to resurrect the notion of programming (however unfashionable that may be), and reject systems which only allow direct manipulation of the interface but are severely limited in their expressive power.

Games are played at the level of superstructure, but to change or rebuild them requires an appreciation of and some access to the platform. It is managing this access — both in terms of what can be seen and what can be done — in order to build a game of one's choice that enables us to help children become aware of how rules operate and the implications of rules. We want to blur the boundaries between user and programmer, between fun at the superstructural level and engagement with the semantics of the platform. In short, we are trying to build a system for children to play with the rules.

Just what we end up building and just what the children learn is a matter of ongoing investigation but at its heart lie many classical questions of microworld design. From all the foregoing discussion, it should be clear that we expect the platform to be intrusive, that is, we anticipate that children will, at some level at least, know what is happening below the superstructural level, understand how to interact with it, and appreciate the mechanism which makes it work. Actually we are coming to believe that the most important appreciation of mechanism is the idea of mechanism — knowing that there are rules which makes things work represents quite a few steps towards understanding precisely how something works.

But wanting children to have access to the platform has raised problems of choice of a platform that affords access to very young children. We decided to base the design and construction of playground on ToonTalk (Kahn, 1999). Its appeal for our purposes is in its "concretizations" of computational abstractions based on the animated metaphors of the computer game. The new concepts of ToonTalk design are twofold: the provision of powerful high-level constructs for expressing programs at the platform level and the provision at the interface level of concrete, intuitive, easy-to-learn, systematic game analogues to every construct.

The fundamental idea behind ToonTalk is that source code is animated. (ToonTalk is so named because one is "talking" in (car)toons.) This does not mean that it takes a visual programming language and replaces some static icons by animated icons. It means that animation is the means of communicating to both humans and computers the entire meaning of a program. Program sources are not static collections of text or even text and pictures, but are animated, tactile, enhanced with sound effects, and clearly physical. The programs of ToonTalk are encapsulated in the actions of robots which are trained by example to perform a role. The conditions, which determine subsequent performance of the actions at run time, can be generalised or specialised after the training has taken place. Details of how ToonTalk works, its design principles and some applications can be found at http://www.toontalk.com/

We are aiming to design our prototype games in ways that make it straightforward to interact both with the objects in the games and the mechanisms behind the objects which give them their behaviours. To achieve this we are engaged in iterative designing and prototyping with children to find the appropriate level of expression for these mechanisms. Unsurprisingly, we have found that if the level is too low, (e.g. raw ToonTalk at the platform level) such young children are typically unable to grasp what is happening; if it is too high (e.g. all the mechanisms are wrapped up in black boxes) the children might be able to play the games, but are not even aware of the rules that make them work, let alone able to inspect and modify.

Our solution to this challenge was to consider the mechanism in levels, thus using and developing the theoretical constructs presented earlier. Figure 2 presents an example of a game and the different levels of mechanism that are accessible to the user. This example will be helpful in both focusing the discussion and allowing us to present some illustrative work with children.

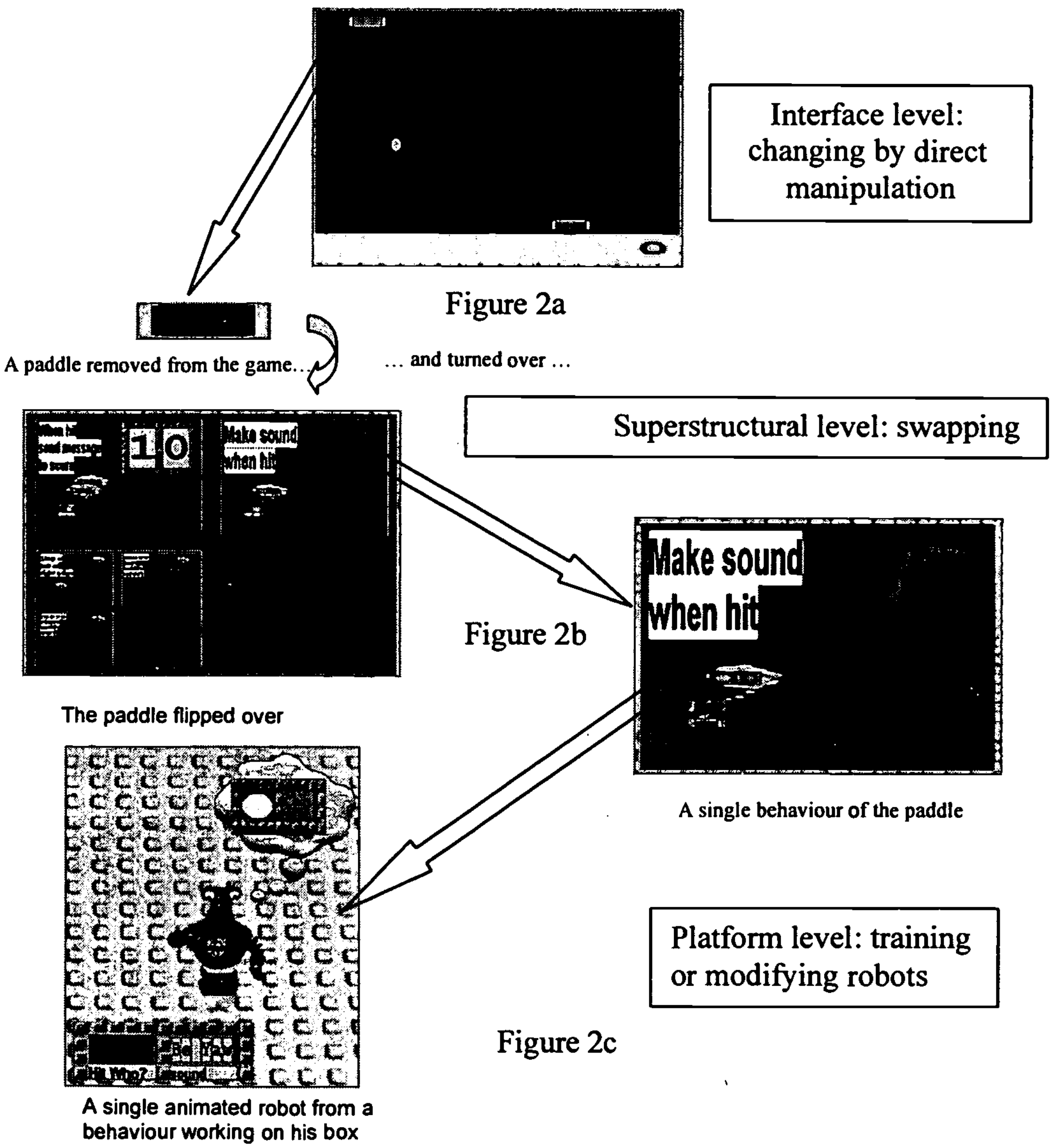


Figure 2: Three levels of mechanism in a Pong game: interface, superstructural and platform

In games, there are a range of components we shall call play objects: balls that bounce and make noises, wizards which turn into frogs, or dice that control a move. Figure 2a presents a simple Pong game that we built in ToonTalk. It consists of a background, two paddles, a ball and a score. There is a scoring connection between the top paddle and the score, so that when the ball is hit by the top paddle the score increases by ten points. Changes can be made at the interface level by direct manipulation and by using ToonTalk tools (for example, there is a simple way to change the colour of the background, or make copies of any play objects using the 'magic wand').

Our effort in microworld design has been to build a superstructural level on top of ToonTalk which allows children to manipulate the 'things that matter' in the game — in this case the *behaviours* of the play objects. We have created a class of playground objects called 'behaviours' which are portable components packaging the functionality of robots into manageable pieces. Functionality for play objects is realised through adding trained robots to their flipside. A key design principle of the behaviours is that we judge them (sometimes correctly!) to be the right grain size for children: that is, they provide an appropriate level of complexity and functionality so that they can be appreciated and used in the design of new objects and rules. Figure 2b shows how flipping the top paddle in Pong exposes its behaviours which comprise robots that '`make sound when hit`' and '`when hit send message to score`'. At present, the descriptions of the behaviours are mainly in the form of natural language (i.e. text), although we are currently adding graphical and audio descriptions of a behaviour, so that its functionality becomes evident in its (dynamic) representation.

At the platform level, as we have said, there are robots dealing with the interactions and events. For example, the robot shown in Figure 2c represents one mechanism belonging to the top paddle. The robot has been trained to play the sound `Be Yaw` when hit by a ball: There are two inputs to this program in the box in front of the robot, and the conditions under which it will act are shown in its thought bubble. These initial conditions are set in the training stage but can be modified subsequently. The '`hit who?`' hole of the input box is a dynamic sensor which shows what is currently colliding with the paddle. In order for the robot to be triggered into action (i.e. its condition satisfied), a picture of a ball needs to appear (this will happen when a ball collides with the paddle). In the second input box, the only requirement is that some sound is present. In figure 2c, the '`hit who?`' sensor is black showing that there is no current collision. However, when the paddle is hit by a ball, all the robot's conditions will be fulfilled and it will play the sound in the second hole of the input box.

We have found that children are able to understand the principles of programming by example, the role of input boxes, the generalisation of conditions etc. However, we have found the robots most useful when used as part of the superstructural level, that is when embedded in behaviours used for carrying out a specific task, such as the behaviour. We shall illustrate this conjecture in the following section.

We are unable in this paper to provide further details of either the platform or the playground/microworlds we are building: these and details of the research objectives can be found at http://www.ioe.ac.uk/playground. Instead, we will illustrate our general line of reasoning by reference to a case study of two children interacting in out Pong microworld.

The relationship between platform and superstructure:

Two girls, Rachel and Heather, both aged 8, started with a two-player Pong game similar to the one in Figure 2a. One of them used the SHIFT and CTRL keys to control the left and right movement of the top paddle, while the other (they took turns) used the mouse to move the bottom paddle. The ball bounced around and the girls each tried to hit it with their paddle. The score (bottom right hand corner) increased by 10 points whenever the top paddle hit the ball and there was also a noise every time the ball hit the top paddle. At this level of playing the game, the mechanisms which drove these actions were largely invisible — but, as we shall see, they were not inaccessible.

At first Rachel and Heather simply treated the game as a closed system during which they noticed that the score was changed by the top paddle only. They invented a new twist — they took turns to play against the clock, trying to get the most points in 30 seconds. However, after a short while they both pronounced the game as 'boring'.

Changes at the superstructural level:

Because of the culture we had developed in our classrooms of changing games, the girls began to think about how they could change the game. The simplest changes they could make involved changing colours and sizes of objects at the interface level.

Heather: "Make it more colourful… it's a bit dark!"

They made the background light blue and the bottom bar brown. (The programming environment allows colour changes to certain objects simply by pointing at them and pressing keys). Interestingly enough, these apparently trivial modifications immediately changed the look and feel of the game, and generated some new suggestions.

Rachel: "[We] could have two scores, one for bottom one for top"

Heather: "…you could have like the paddle as a fish"

Rachel: "I've got an idea … Bammer hits the thing down and hits the ball."

Rachel's idea was that the ball should be changed into a picture of Bammer the mouse, one of the creatures who inhabit the ToonTalk platform (Bammer's jobs include adding numbers, merging pictures and concatenating strings of text). The new object (a picture of Bammer) needed to retain the functionality of the old object (the paddle) but have a different face. This, Heather and Rachel realised, could be achieved by transferring all the behaviours — something the girls knew how to do.

Heather: "I know — you stick the paddle on the back."

The girls transferred the functionality of the paddle to a picture of Bammer, by turning both over and placing one on the other. This operation at the superstructural level has platform level functionality because of ToonTalk's object oriented design. Similarly the behaviour objects on the back of the paddle transfer their functionality, giving Bammer the right behaviours. This is illustrative of the delicate relationship that has to exist in our playground design: while the platform provides the means to effect the behaviour transfer, it is having the right things in the right place at the superstructural level which enables the children to make use of this functionality.

The two girls could effect other changes at this intermediate level. They changed the ball to a bird and transformed all its functionalities to give the game the appearance shown in Figure 3, in which paddles and ball are replaced by bammers and bird.

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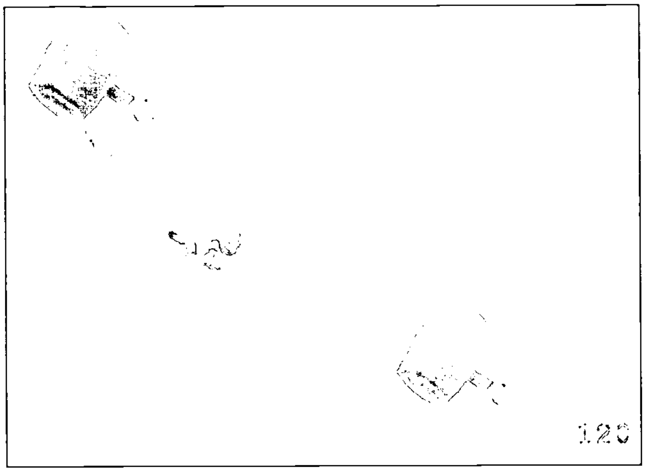

Figure 3: Pong with changed appearance but same functionality

Now the changes in colour stimulated more ideas as it supported the girls' inclination to build an underwater narrative: watching this, we suddenly recalled that Rachel had mentioned earlier that she wanted to change the paddle to a fish!

Rachel: "I know that's like the sea and he's [Bammer] running down into it! Coz that's like there's a hill and there's sand going down."

Heather: "There's a problem! He's walking on… the water!"

Rachel: "It doesn't matter"

The new game was structurally very similar to the original, but to the girls it had suddenly become far from boring! On the contrary, it was now a compelling game, not least because they had made it themselves.
In the next session, we gave the girls some new pictures of fish and sharks to help them in their objective of making their game 'go underwater'. They made two copies of the shark picture — one for each of the paddles, and discussed further changes: Rachel wanted to have lots of fish bouncing up and down, an idea she had picked up from other children who had made multiple balls in their games. They then started to change the paddles to sharks, an easy enough task involving essentially the same procedure of behaviour transfer as they had used in changing the paddle into Bammer. At this point, they similarly changed the ball to a fish and made copies of it to place underwater (see Figure 4).

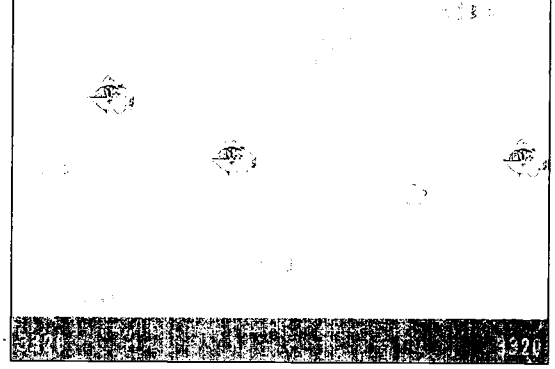

Figure 4: Sharks, fish and two copies of the same score

How much programming were Heather and Rachel doing? It is tempting to say that at this point, they were simply replacing pictures with other pictures. But in the process, they exposed the static representation of the programming platform. While the grain size of their actions was large when programming at the behaviour level, a finer grain size was made visible —the mechanisms were visible even if they remained intact. Heather and Rachel were also familiar with robots and had programmed at this level in some simple cases. They therefore had an idea of what it was that was inside the behaviours and caused them to work.

At the beginning of the session only the top shark scored points — as in the original Pong game. The girls wanted to make their game competitive by adding the same functionality to the bottom paddle, so they could play against each other. They copied the score object on the bottom right of the game so it also appeared on the left (see Figure 4). They also realised that they had to add some functionality to the bottom

shark so copied the behaviour on the top shark (as shown in Figure 2b) and put it on the flip side of the bottom shark. These two changes, the first at interface level and the second at the superstructural level, revealed a bug in their thinking about the mechanism of the scoring system and required an intervention at the platform level as we shall describe later.

Changes at the Platform level:

The sight of the sharks in an underwater context with fish, provoked the two girls to make more suggestions for changes:

Rachel: "The sharks are the paddles. And if one of those hit the sharks- any of them …"

Heather: "it goes like this … 'chomp'!"

They wanted a different sound that played whenever the sharks hit the fish. At this point, only the top shark had a sound behaviour (left over from being a paddle in the Pong game). To change the sound they removed the behaviour labelled (see Figure 5) and dug down towards the platform level to investigate the mechanism. They found that the robot was trained to play the sound ' every time its conditions were satisfied. So in order to reconfigure this behaviour they had to remove ' and replace it with ' . This they managed easily, demonstrating how the two children interacting at superstructural level could, in simple cases when it mattered to them, move into the platform and modify the program.

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While playing this new version, Heather controlled the top shark and Rachel the bottom one. It took them some while to notice that both scores 'belonged' to both sharks and were always the same.

Heather: "You see! I said we shouldn't have copied it!"

Heather knew there was a problem and explained again that they were planning to have a *separate* score for each shark:

Heather: "We want to…change it so that each one has its own score… so each shark has its own score."

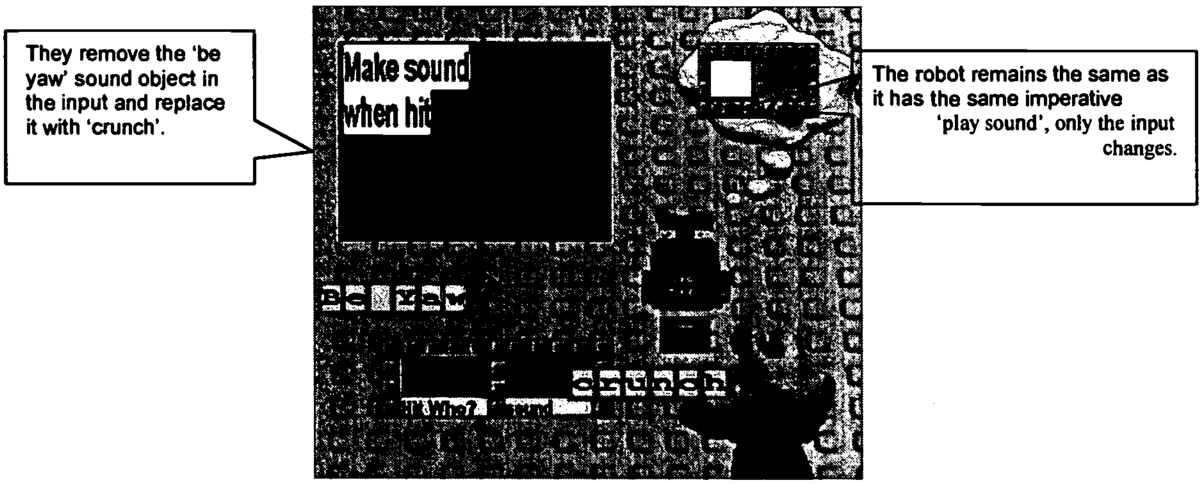

Figure 5: The robot that makes a sound when hit

There had been a bug in their thinking, but to fix it they had to find out more about the mechanics of the score system: they needed to see the nature of the connection between the sharks and the scores. This is quite a complicated business. Any object oriented language needs a mechanism by which objects can send messages to each other — this is one of the powerful ideas of object oriented programming. ToonTalk is no exception. But it is different in one important respect — the message passing is instantiated in a concrete metaphor; quite simply, birds take messages to nests, (see Figure 6) so by programming a robot to give something to a bird whose nest is on another object, a message is passed to that object and the bird returns to its starting place. This gives a one-to-one communication channel.

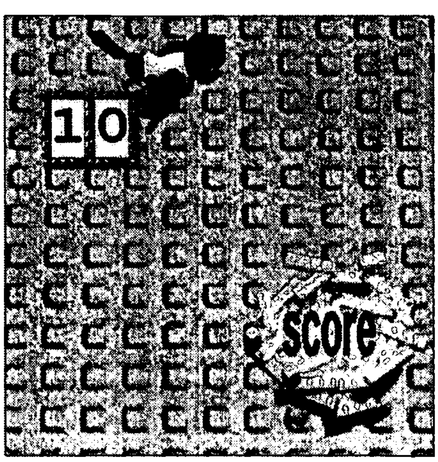

Figure 6: A bird takes a message to its nest: the communication metaphor in ToonTalk

Heather and Rachel took off the top shark and looked again at its behaviours by flipping the picture over. They found the scoring behaviour, `hit picture send message to score` and took it out. We explained that the robot had been trained to give the number 10 to the bird each time the shark hit a picture. This 10 then 'magically' was added to the score. How did it get there?
In the case of the scoring mechanism, the bird in the `hit picture send message to score` behaviours on a shark had a corresponding nest in a behaviour on a score. An understanding of this platform level metaphor would be crucial if the girls were to implement their competitive shark game successfully.
To make the platform visible in a dynamic way, we suggested that the girls took off one score and put it next to the game. The game worked as before, until someone scored. At this point, the score was incremented: but the mechanism of the increment had now become visible — when either shark was hit a bird flew out and delivered the message '10' to the score, accompanied by dramatic wing-flapping sound effects (all this is part of the animation of the platform).
When the second score was removed and placed next to the game, the visibility of the scoring mechanism was even more dramatic. If the bird has been copied, both birds fly to the same nest giving many to one communication. This essentially is what had happened when the girls copied the
behaviour. But if the nest is also copied (as the girls did when they copied the score),

the bird makes a copy of itself and the message — two identical messages are delivered, one to each nest (one-to-many). We have tried to illustrate this in Figure 7, which shows two birds emerging from the flip side of the bottom shark after it had been hit by the ball.

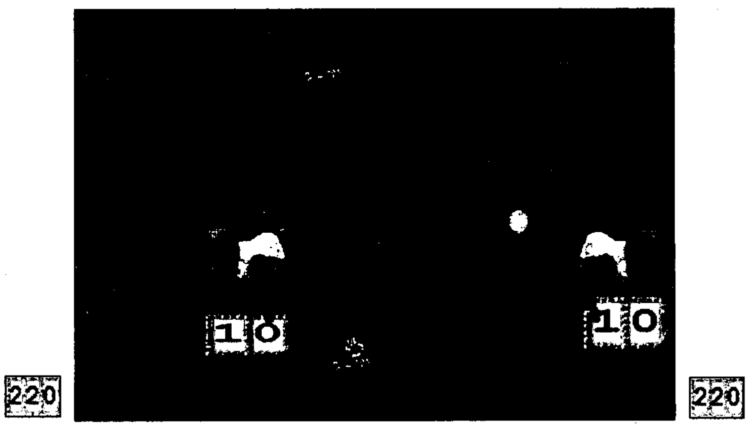

Figure 7:Two identical birds flying to their different nests

After the children had played their new game and watched the birds delivering their scores, we decided to check if the metaphor was transparent:

Researcher: "What do you think is going to happen, Heather, when we start the game?"

Heather: "Both sharks are going to have numbers coming out of them."

Researcher: "Where are they going to go to?"

Rachel: "Into the Points."

Now the girls could see two birds fly out each time a shark was hit, one bird flying to each score. Although still playing the game, a change at the superstructure level (removing the scores) had led to an exposure of the platform, an interplay between superstructural and platform levels. At the simplest level, a mechanism built into the platform revealed the bug in the girls' game design and showed Heather and Rachel why the scores were not responding as they wanted them to.
At a deeper level, revealing the mechanism in this way gave an entry into the bigger ideas of using the generative power of programming. Constructing further insight into the mechanism now means another connection into the scheme of the code that can be used for construction later.
Finally, to complete the description of the evolution of Heather and Rachel's game, we explained that if two separate scores were required we needed to have one bird per shark. We helped them replace the bottom shark-score connection with a new bird-nest pair. Their two-player game was now complete as illustrated in Figure 8.

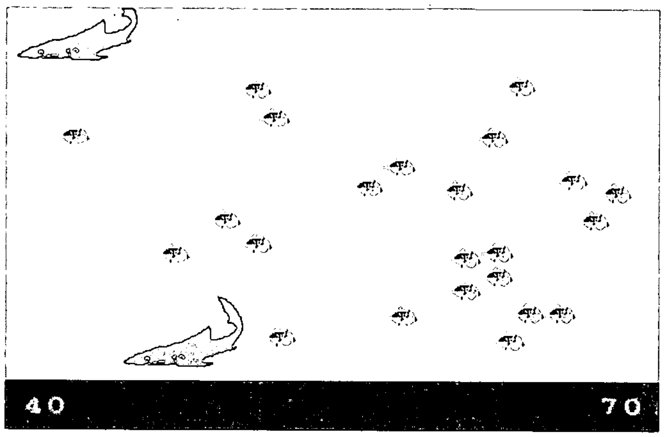

Figure 8: Shark game with two separate scores

Discussion and Conclusions:

Let us compare the mathsticks and playground ideas. Comparing Heather and Rachel's activities with the Mathsticks microworld, the most obvious surface-level change is in the substitution of directly manipulable (animated) elements for text-based programs. This difference is, however, a symptom of evolutionary change. The key difference, we believe, is not only in the new kind of language (certainly important) but also in the interplay between superstructure and platform.
The search for replacements to old kinds of formalism based on textual strings is certainly going to be a defining difference of the new century's mathematical expression (see Kaput, 1994, for a thought-provoking discussion). In general, its potential lies in the broadening of expressive power, to include more immediate, graphical, dynamic and expressive entities and the exposure of relationships between them. In programming terms, therefore, this will allow a richer set of metaphors which provide mappings from abstract computational entities and actions to the concrete world of objects, sounds and even gestures.
Using ToonTalk as a platform has shown us that even quite young children can recognise that they can build objects from scratch (by training robots), combine, generalise and debug. More generally, they can come to think about mechanisms — how things work, why they work, how they can be rebuilt. In the twenty-first century,

where the opportunity to strip down working systems with spanners and screwdrivers are much more limited than they were, we will need to consider virtual alternatives, ways for children to take things to pieces, look at what makes them tick, and put them back together (see Noss, 1998, for a discussion of the implications of this view for mathematical learning).

There are also prices to be paid. While our case study illustrates the expressive power of a non-textual programming language, and the opportunities it affords for building superstructures which are appealing and powerful, the absence of a textual description frequently renders the programming to be cumbersome and time-consuming for designers and seriously limits communicability to peers and teachers. This latter issue resonates with the heated debates over the advantages and disadvantages of direct manipulation and text-based interfaces for the learning of mathematics (see, for example, diSessa, Hoyles, & Noss, 1995 for a report of one such debate in the context of dynamic geometry): proponents of the former point to a sense of engagement developed with screen objects; advocates of the latter stress the importance of a language for description, reflection and communication. Perhaps our criticism at that time of direct manipulation missed an essential point — that point and click is precisely the right mechanism for building an expression of the relationships, but precisely the wrong one for reflecting on it and communicating it. There is a duality between expression and reflection, and we must find new ways to play one off against the other. We are currently undertaking a more controlled comparison between ToonTalk-based and Logo-based playgrounds, which will allow us to explore the relative strengths and weaknesses of the two approaches, hopefully leading us to design more successfully in both.

Our superstructural level of behaviours seem to have hit on a way to offer young students some of the power and manipulability of programming and, at the same time, some awareness of the mechanisms of the platform. They are of about the correct grain-size for children: But they also open a window on to deeper issues. The different representations of the behaviours afford a means to think about a rule, what makes it true, and the limits of its validity (e.g. is A hits B the same as B hits A?). In our case study we described the simple rule '`make sound when hit`' which later became transformed into the more complicated

Our contention is that by looking at how these mechanisms work, and changing them, children are better able to be appreciate inference and conditionality and even make these relationships explicit.

In retrospect, it seems that these metaphors of grain size, levels and permeability between levels are crucial for incremental learning. Perhaps this is the reason why there are not many examples of computational worlds which afford interesting and creative directions for children to learn mathematical ideas *and* which provide an entrée into the world of formal systems which child-programming had always claimed to offer: the set of tools and metaphors appropriate for navigating around at the interface level were *not* functional below that level — to get below, one had to enter a new world of arcane (usually textual) difficulty. Reflecting on our earlier position, we see that we identified platform with text. Now we can explore how new platforms offer new opportunities which preserve what is essential about microworlds, but which increase the functionality at the programming level for learners.

Our purpose is not to claim that we have found a 'solution'. In any case, there are new difficulties emerging: we currently have no mechanism for abstraction — which is combining a collection of programs into a new program and naming this. And the directly manipulable animated elements of programming robots come at the price of no sensible editing mechanism. The case of Heather and Rachel is, however, indicative of possible futures for microworld design. Exposing the birds to reveal the mechanism of message passing feels like an example of a more general possibility: ought we not to be thinking about building systems more generally whose mechanisms are visible and accessible at some level? Mathsticks showed us the importance of linking actions, visual and symbolic representations whilst maintaining the symbolic as a programming tool.

The playground example showed us something more: by redefining what we mean by the representation of mathematical process and object, it opens doors into new kinds of mathematical (and scientific) epistemologies for children. In this respect, the technology on which it is built certainly does point in new and interesting directions which will — if schools and systems allow it — redefine what is meant by mathematical and scientific learning in this new century.

References:

Abelson Harold, Gerald Jay Sussman and Julie Sussman (1985) *Structure and interpretation of computer programs* Cambridge, Mass.: MIT press.

Brousseau, G. (1984). The Crucial Role of the Didactical Contract in the Analysis and Construction of Situations in Teaching and Learning Mathematics. In H. Steiner (Ed.), *Theory of Mathematics Education* (Vol. 54, pp. 110-119). Bielefeld: Institut für Didaktik der Mathematik (IDM), Universität Bielefeld.

Clements, D. H. & Sarama, J. (1995). Design of a Logo environment for elementary geometry. Journal of Mathematical Behaviour, 14, 381-398.

Clements, D. H., Battista, M. T., Sarama, J. & Swaminathan, S. (1996). Development of turn and turn measurement concepts in a computer-based instructional unit. *Educational Studies in Mathematics*, 30, 313-337.

diSessa, A. & Abelson, H. (1986). Boxer: A reconstructible computational medium. *Communications of the ACM*, 29/9, 859-868.

diSessa, A. (1985). A Principled Design for an integrated Computational Environment. *Human-Computer Interaction*, 1, 1-47.

diSessa, C., Hoyles, C. & Noss, R. (1995). *Computers and Exploratory Learning*. NATO ASI Series, Subseries F, 146. Heidelberg: Springer-Verlag.

Edwards, L. D. (1992). A Logo Microworld for Transformation Geometry. In L. Hoyles & R. Noss (Eds.), *Learning Mathematics and Logo* . Cambridge, Massachusetts: MIT Press.

Edwards, L. D. (1995). Microworlds as Representations. In A. A. diSessa, C. Hoyles & R. Noss (eds) *Computers and Exploratory Learning*, 127-154. Berlin/Heidelberg: Springer-Verlag.

Eisenberg, M. (1995). Creating Software Applications for Children: Some Thoughts About Design. In A. A. diSessa, C. Hoyles & R. Noss (eds) *Computers and Exploratory Learning*. Berlin/Heidelberg: Springer-Verlag, pp.175-196.

Harel, I. (1988). *Software design for learning: children's constructions of meanings for fractions and logo programming*. Unpublished doctoral dissertation. Cambridge MA: MIT Laboratory

Healy, L. & Hoyles, C.(2000) A Study of Proof Conceptionsin Algebra *Journal for Research in Mathematics Education*, 31, 4, pp 396-428

Healy, L. & Hoyles, C. (1999). Visual and Symbolic Reasoning in Mathematics: Making connections with computers? *Mathematical Thinking and Learning*, 1, 1, 59-84.

Hoyles, C. & Healy, L. (1999) Linking Informal Argumentation with Formal Proof through Computer-Integrated Teaching Experiments. In the *Proceedings of the Twenty-third International Conference for the Psychology of Mathematics Education* Haifa, Israel.

Hoyles, C., & Noss, R. (1987a). Children Working in a Structured Logo Environment: From Doing to Understanding. *Recherches en Didactique des Mathématiques, 8*(1.2), 131-174.

Hoyles, C., & Noss, R. (1987b). Synthesising Mathematical Conceptions and their Formalisation through the Construction of a Logo-based School Mathematics Curriculum. *International Journal of Mathematics Education in Science and Technology, 18*(4, July/August), 581-595.

Hoyles, C., & Sutherland, R. (1989). *Logo Mathematics in the Classroom*. London: Routledge.

Hoyles, C. & Noss, R. (1996). *Windows on Mathematical Meanings: Learning Cultures and Computers*. Pp. 275. Netherlands: Kluwer Academic Press.

Hoyles, C. (1993). Microworlds/Schoolworlds: The Transformation of an Innovation. In C. Keitel & K Ruthven (eds) *Learning From Computers: Mathematics Education and Technology*, 1-17. Berlin/Heidelberg: Springer-Verlag, pp. 1-17.

Kafai, Y.B. (1995). *Minds in play: Computer game design as a context for children's learning.* Lawrence Erlbaum Associates.

Kahn, K. (1999). Helping children learn hard things: computer programming with familiar objects and activities *in* Druin, A. (ed) *The design of children's technology.* San Francisco: Morgan Kaufman Publishers Inc. 223-241.

Kalas, I. (1997) Thomas the Clown. University of Comenius, Slovak Republic.

Kaput, J. J. & Roschelle, J. (1999). The Mathematics of Change and Variation from a Millennial Perspective: New Content, New Context. In C. Hoyles, C. Morgan & G. Woodhouse (eds) *Rethinking the Mathematics Curriculum*. London: Falmer Press, Chapter 12, Section 2, pp. 155-170.

Kaput, J. (1994). Democratizing Access to Calculus: New Routes using Old Roots. In A. Schoenfield (ed) Mathematical Thinking and Problem Solving, Erlbaum, Hillsdale, NJ, pp. 77-156

Noss R. (1998). New Numeracies for a technological culture. *For the Learning of Mathematics*, 18, 2, 2-12. 1998

Noss, R. and Hoyles, C. (1996) *Windows on Mathematical Meanings: Learning Culture and Computers*. Dordrecht: Kluwer.

Noss, R., Hoyles, C. & Healy, L. (1997). The Construction of Mathematical Meanings: Connecting the Visual with the Symbolic. *Educational Studies in Mathematics*, Special Edition, 33, 2, 203-233.

Papert, S. (1991). Situating Constructionism. In I. Harel & S. Papert (eds) *Constructionism*. New Jerseyt: Ablex Publisshing Corporation. 1-12.

Thompson, P. (1992). Notations, Conventions, and Constraints: Contributions to effective uses of concrete materials in elementary mathematics. *Journal for Research in Mathematics Education*, 23/2, 123-147.

Turcsányi-Szabó, M. (1999). Logo Connections: Some cunning aspects. In R. Nikolov, E. Sendova, I Nikolova & I Derzhanski (eds.) *Proceedings of the Seventh European Logo Conference*, 80-91. Sofia, Bulgaria.

Educating students for their future not our past: a challenge for teachers of mathematics[4]

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Introduction:

We have all benefitted from the industrial revolution which began in the 18th century and which saw the gradual replacement of manual labour by machine. In the late twentieth century, we are going through another revolution brought about by the rapid development of increasingly cheap and powerful computer based technologies. Whereas the industrial revolution occurred through the mechanisation of manual labour, the current electronic revolution is being achieved through a mechanisation of certain sorts of *intellectual* skills. As stated by Atiyah (1986, p. 43), 'It is the brain rather than the hand that is being made redundant'.

Initially, from the mathematical point of view, the sort of intellectual skills that could be mechanised on an everyday basis might be dismissed as being relatively elementary, in nature. For example, the performance of routine arithmetic operations through the use of adding machines and more recently handheld calculators. However, increasingly, we are entering a world in which many more of what were previously considered to be high level mathematical skills, for example the ability to manipulate complex algebraic expressions, are now also capable of being mechanised on an everyday basis through the emergence of handheld computer algebra systems (CAS).

How should we as mathematics teachers react to this ever increasing mechanization of intellectual skills, the teaching of which is our 'bread and butter'? One approach is to do as the Luddites did, destroy the machines: in modern terms, ban the technology from the classroom as is currently happening in California (see for example, Becker and Jacob, 1998) and, in the process, continue to prepare students for the world of our past. The other approach is to realize that, while the emerging technologies do appear to threaten to make much of what we taught in the past redundant, they also offer real opportunities to enhance the mathematical capabilities of our students (see for example, Ralston, 1999). They have done so in the past and they will continue to do so in the future. What is so difficult for most of us at present is both the pace and the scope of the change. For many of us, it is almost equivalent to experiencing a 'thousand' years of change in a professional life time, as we can see by tracing technological developments in the mathematics classroom over the past 30 or so year.

[4] An earlier version of this paper was presented at ICME 5 (Jones, 1996)

Looking back :

If a 11 year old student in the 1960's was asked to evaluate 234 □346, they would have most likely reached for a pencil and paper, written the two numbers down on the paper, one above the other, and then performed the process of a long multiplication algorithm in a similar manner to that shown below:

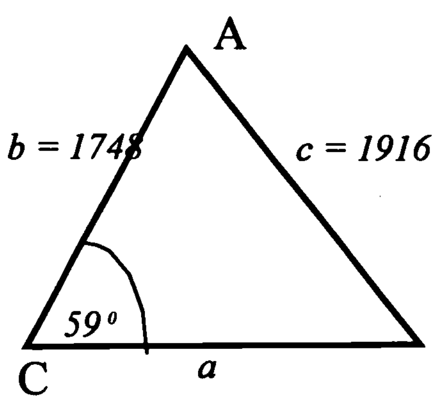
$$\begin{array}{r} 234\\ \underline{346}\\ 1404\\ 936\\ \underline{702}\\ \underline{80964}\end{array}$$

This is not a technology free process. In carrying out this computation, the student has used a technology, pencil and paper, to record the numbers to be multiplied. This in turn frees her mind to perform a series of mental operations on these numbers, the results of which are recorded sequentially for subsequent processing. Few students of that era were trained to carry out such computations without relying on such technology. On moving into secondary school, our student was required to move beyond whole number arithmetic and learn how to operate on numbers involving decimals. To assist her in that task she would have been introduced to a new technology, the table of logarithms. Tables of logarithms enabled complex arithmetic expressions involving decimal numbers to be transformed into less complex sums, differences or simple multiples, which could be systematically evaluated with the aid of pencil and paper as shown below.

number	logarithm
2.34	0.3692
0.0346	$\bar{1}.4609$
0.08096	$\bar{1}.0917$

Up until the 1970's, the table of logarithms was the only computational technology routinely available to students in the classroom and its role was pivotal in certain areas of the mathematics curriculum, as can be seen from the worked example of an application of the sine rule taken from a 1960's mathematics textbook (Rose, 1964).

Example 14. - Solve the ΔABC completely when $c = 1916$ ft., $b = 1748$ ft. and $C = 59°$.



To find B – $\dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

and hence – $\sin B = \dfrac{b \sin C}{c}$

Taking logs throughout –

$$
\begin{aligned}
\log \sin B &= \log 1748 + \log \sin 59° - \log 1916 \\
&= 3.2425 + \bar{1}.9331 - 3.2823 \\
&= \bar{1}.8933 \quad = \log \sin 51° 28' \\
B &= \underline{51° 28'}
\end{aligned}
$$

$$
\begin{aligned}
\text{Then} - \quad A &= 180° - (59° + 51°28') \\
&= \underline{69° 32'}
\end{aligned}
$$

This example clearly illustrates the pivotal role played by logarithmic computations in solving such problems. Without these computational skills, such problems could not be solved, at least in the classroom. Unfortunately, for many teachers, the need to acquire such skills became an end in itself, rather than a means to an end, only made necessary by the lack of a practical alternative. Once the electronic calculator became common place in the classroom, the need for tables of logarithm for computations became unnecessary. Yet, at a workshop conducted for teachers on the use of the first electronic scientific calculators in the early seventies (Barling, 1995), one of the reasons given to teachers for introducing the calculator into their classroom was that it would obviate the need for students to use logarithm tables and would considerably speed up the process. It was stated that the calculator could be used to generate the logarithms, do the additions and then take the antilogarithms to obtain the required answer!

Why do such things happen? In part, it is due to the general lack of recognition that mathematics, like all human intellectual activity is always shaped by the available technology, but that, with time, the technologies 'become so deeply a part of our consciousness that we do not notice them' (Pea, 1993, p. 53). As a result, the technology effectively becomes 'invisible', while the activities it generates can come to be seen as mathematical activities in their own right, for example, carrying out calculations using logarithms. Hence, when a new technology such as the electronic calculator is introduced, it is common for it to be promoted as a means of 'enhancing' the teaching of such activities, even though the technology itself has been designed to obviate the need for such calculations. Kaput (1992, p.548) has termed this phenomenon 'retrofitting'. The irony of using a technology such as a calculator to help complete computations with logarithms should not be lost on anybody. Yet today, with CAS equipped graphics calculators (for example, the TI-89) now beginning to find their place in the everyday mathematics classroom in a number of countries, the suggestion that CAS might be used to help improve students' pencil and paper algebraic manipulative skills has a similar ring.

The tendency for aspects of mathematical practice to becomes an end in themselves rather than a means to an end, is clearly illustrated in modern calculus texts. For example, let us say that we wish to calculate the length of the arc of the curve $y = \ln x$

between $x = 1$ and $x = \sqrt{3}$. The solution reproduced below is based on a solution given in a typical calculus text (Grossman, 1977).

SOLUTION. Here $f'(x) = \dfrac{1}{x}$ so that

$$s = \int_1^{\sqrt{3}} \sqrt{1 + \frac{1}{x^2}}\,dx = \int_1^{\sqrt{3}} \frac{\sqrt{x^2+1}}{x}\,dx$$

Let $u = \tan\theta$ so that

$$s = \int_{\pi/4}^{\pi/3} \frac{\sec\theta\sec^2\theta}{\tan\theta}\,d\theta = \int_{\pi/4}^{\pi/3} \frac{\sec\theta(1+\tan^2\theta)}{\tan\theta}\,d\theta$$

$$= \int_{\pi/4}^{\pi/3} \left(\frac{\sec\theta}{\tan\theta} + \sec\theta\tan\theta\right)d\theta = \int_{\pi/4}^{\pi/3} \left(\frac{1}{\cos\theta}\frac{\cos\theta}{\sin\theta} + \sec\theta\tan\theta\right)d\theta$$

$$= \int_{\pi/4}^{\pi/3} (\csc\theta + \sec\theta\tan\theta)\,d\theta$$

$$= \left(\ln\left|\csc\theta + \cot\theta\right| + \sec\theta\right)\Big|_{\pi/4}^{\pi/3}$$

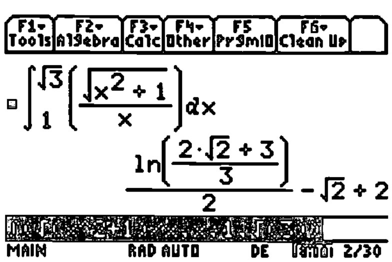
$$= \left\{\left[-\ln\left(\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right) + 2\right] - \left[-\ln\left(\sqrt{2}+1\right) + \sqrt{2}\right]\right\}$$

$$= \ln\left(\sqrt{2}+1\right) - \ln\sqrt{3} + 2 - \sqrt{2} = \ln\left(\frac{\sqrt{2}+1}{\sqrt{3}}\right) + 2 - \sqrt{2}$$

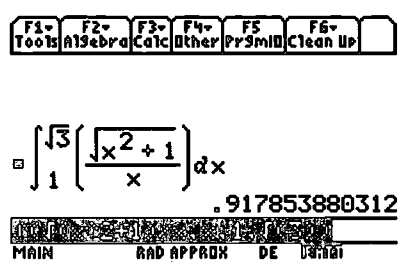
In looking at this solution we see that it bears an uncanny similarity to the 1960's textbook solution to the sine rule problem. First, some theoretical knowledge is used to set up the solution to the problem with pencil and paper. In the case of the sine rule application this results in a complex arithmetic expression which is then evaluated with the aid of log tables. In the arc length problem the solution is set up in the form of a definite integral which is then evaluated using an appropriate substitution and some algebraic manipulation to enable the original integral to be transformed into standard form. The results of the manipulation are recorded with pencil and paper and presumably a table of standard integrals is used in the end to help evaluate the resulting integrals. From figure 2 we can see that the arc length is

$$\ln\left(\frac{\sqrt{2}+1}{\sqrt{3}}\right) + 2 - \sqrt{2} = 0.9178538803.$$

However, if we have access to a graphics calculator with symbolic processing capabilities like the TI-89, we can simply utilize the integration facility to obtain the answer in exact form or in approximate numerical form, see figure 1.



Exact form



Approximate form

Figure 1: Using a TI-89 to evaluate the integral to obtain the arc length in both exact form and approximate form.

For most of us who learnt calculus as a pencil and paper based activity, it would be hard to accept that the steps involved in evaluating the definite integral in the arc length problem are not worthwhile mathematics, yet, if the true purpose of the activity was to evaluate the arc length, then the process as a whole may have no more intellectual value to the majority of students than the mastering of the skills needed to carry out complex arithmetic computations with tables of logarithms. Just as the electronic calculator was designed to avoid the need for human beings to carry out complex arithmetic computations by hand, a graphics calculator with numerical integration capabilities is designed to avoid the need for human beings to, amongst other things, evaluate complex definite integrals. This is challenging to those of us for whom the only technology supporting our calculus activities was pencil and paper and possibly tables of standard integrals. We had to master integration methods to solve more advanced problems, just as students in the past had to master computations with logarithms to solve more advanced mathematical problems. Thus we see that the available technology is a prime determinant of what mathematics we do in the classroom and how we do it, both now and in the past. So what is different now?

Intelligent technology:

To explore this question at a level that enables us to go beyond the specific technology involved, we need to recognise that the technologies we have used to support the teaching and learning of mathematics in the classroom, both now and in the past, can be regarded as 'intelligent' in the sense that they can 'undertake significant cognitive processing on behalf of the user' (Salamon, Perkins & Globerson, 1991. p. 4). Even pencil and paper, when used to support mathematical activity, can be regarded as intelligent technology. For example, when carrying out algebraic manipulations using pencil and paper, we record the results of intermediate steps so that we do not have to keep these results in our working memory at the same time as we carry out the mental processes involved in the manipulation. Thus pencil and paper can be regarded as being intelligent in that we use it to share the cognitive load when carrying out algebraic manipulations. Similarly, the use of a table of standard integrals shares the cognitive load of evaluating a complex integral by reducing the information we need to keep in working memory or retrieve from long term memory whilst carrying out the intermediate steps in the process.

If the older pencil and paper based technologies can be regarded as intelligent, what then differentiates them from the newer computer based intelligent technologies? Whereas the older technologies can share the cognitive load by acting as storage devices, computer based technologies not only store information but also have the added dimension of being able to carry out significant processing of that information with minimal intellectual input from the user. For example,. a graphics calculator with symbolic processing capabilities can store an algebraic expression but then, on command, carry out a variety of algebraic processes of the sort that would have required considerable mental effort on our behalf when working with pencil and paper only. The ability of computer based technology to both store and process mathematical information significantly increases the potential to share the intellectual burden with the user. However, computer based technology cannot plan, model, synthesise, interpret, etc. At present, these are intellectual abilities possessed only by burden with the user. However, computer based technology cannot plan, model, synthesise, interpret, etc. At present, these are intellectual abilities possessed only by the human mind, which can, of course, also store information and carry out rule based processing. A schematic view of the differing intellectual capabilities of pencil and paper based technology, computer based technology and the human mind is shown in figure 2. paper based technology

information storage

computer based technology

information storage	rule based processing

human 'technology'

information storage	rule based processing	modelling interpretation etc

Figure 2:Schematic overview of the differing intellectual capabilities of pencil-and-paper based technology, computer based technology and the human mind

The higher level thinking skills are the skills that we ultimately value in mathematics but, in practice, we spend most of the time teaching and developing processing skills, particularly algebraic manipulative skills. In part this is because, in a pencil and paper based classroom, mastery of these skills is a necessary prerequisite to using mathematics at a higher intellectual level. Unfortunately, because of the time taken and the intellectual effort needed to develop such skills, the greater part of classroom instruction has been devoted to the acquisition of these skills. As a result, the mastery of these skills has become the primary goal of the majority of mathematics classrooms, and mastery of these skills has become equated with mathematical ability. Again, the means have become the end. Thus any technology that appears to enable a person to carry out such tasks at the push of a button challenges our traditional concept of what constitutes mathematical ability. However, this is only a problem if we continue to view mathematical intelligence as residing entirely within the individual. As we will see, it also limits our thinking about the potential educative role of technology in mathematics.

Intelligent partnership and mathematical ability:

What are the educational consequences of thinking of the technology we use to support mathematical as being 'intelligent'? One is the potential for the development of what has been termed an *intelligent partnership* . In an intelligent partnership, the potential exists for intellectual performance of the partnership to be 'far more "intelligent" than the human alone' (Salamon et al, 1991, p. 4). For example, with access to technology such as a graphics calculator, students have the potential to pursue graphical methods of solution and analysis that greatly exceed what they could ever hope to achieve with a pencil and paper alone, even in principle.

This possibility of students forming intelligent partnerships with technology in mathematics gives them the potential to work at a level in mathematics that may be totally unachievable without the technology. This, in effect, calls into question our traditional notions about what constitutes mathematical intelligence and how it should be assessed. Should it be measured by the mathematical performance of the student working without any technological aid, or does the possibility now arise of it being also recognised as the mathematical performance of a joint system? If we accept that a student working in an intelligent partnership with computer based technology is a legitimate and valued form of mathematical activity, then we must consider the possibility that appropriate assessment of mathematical intelligence involves assessment of that partnership. Further, given that, in the long run, almost all real mathematical activity involves the use of some supportive computer based technology, it could be argued that one of our prime pedagogic interests in mathematics should be directed at the task of developing instructional strategies for building and assessing the mathematical intelligence of such partnerships and not just the individual working alone.

Unfortunately, intelligent partnerships do not appear to be self generating and the challenge for teachers is to develop instructional strategies that promote their formation. And, more importantly, it is unlikely that they will be realised unless students have the same sort of access to the necessary technology as they currently have to pencil and paper. In this regard, handheld technology such as the graphics calculator is likely to have far greater potential than a computer as it is cheap enough and small enough to be in the hands of students at all times. Finally, there is also a need to reassess what is taught, as the knowledge and understandings needed to develop an intelligent mathematical partnership when working with technology are almost certain to differ in some significant ways from those needed for students who will do all their mathematics without access to technology.

Summary and conclusion:

In this paper I have argued that when thinking about the role of the newer hand held computer based technologies in the mathematics classroom we first need to realise that we have always used technology to support mathematical activity in the classroom but, because of its familiarity we have not been very good at separating out what is mathematics in its own right and what is only of value because of the technology we have at our disposal. As a consequence, whenever a new and different technology emerges there has been a natural tendency to retrofit the new technologies to the mathematics activities with which we have become most familiar without any real regard for their relevance in the new technological environment. While this retrofitting of the technology has superficially appeared to bring about significant pedagogic gains in that it enhanced the learning of skills previously difficult to teach, such uses of the new technologies activities are more often than not of transitional value (see, also, Kaput, 1992, p. 517). Secondly, we need to recognize that the technologies we have used to support the teaching and learning of mathematics in the classroom, both now and in the past, can be regarded as 'intelligent' in that they have the ability to reduce cognitive load. However, the new computer based technologies are qualitatively different from the older pencil and paper based technologies because of their ability to both store and process mathematical information. Finally, in recognizing the 'intelligent' nature of the technology we open up the potential for the formation of intellectual partnerships which have the potential to be far more mathematically intelligent than human intelligence alone. This, in effect was what we were aiming for when the technology of the mathematics classroom was pencil and paper based, but we failed to recognize this because the intellectual potential of these technologies was far less obvious than that of the newer technologies. From this point of view, the goals of mathematics education are not under challenge. What is under challenge is the means by which we try to achieve these goals.

References:

Atiyah, M, (1986) Mathematics and the Computer Revolution. In A.G. Howson & J.P. Kahane (Eds.). *The Influence of Computers and Informatics on Mathematics and its Teaching*. Cambridge: CUP.

Barling, C. R. (1995) Personal communication.

Becker, J. and Jacob, B.(1998): 'Math War' Developments in the United States (California), ICMI Bulletin, 44 , 16-25.

Grossman, S. I.(1977) *Calculus* Academic Press: New York

Jones, P. L., (1996) Handheld Technology and Mathematics: Towards the Intelligent Partnership. (Unpublished).

Kaput, J. J. (1992) Technology and mathematics education. In D. A. Grouws (Ed.), *Handbook of research in mathematics teaching and learning*. MacMillan, New York.

Ralston, A. (1999) Let's us Abolish Pencil-and-Paper Arithmetic. *Journal of Computers in Mathematics and Science Teaching.* 18, Number 2, 173-194.

Pea, R. (1993) Practices of distributed intelligences and designs for education. In Salamon, Gavriel (Ed.) *Distributed cognitions: Psychological and educational considerations*. Cambridge University Press: New York.

Rose W.N. (1964) *Mathematics for Engineers Part 1* Ninth Edition Chapman & Hall Ltd: London.

Salamon, G., Perkins, D.N., & Globerson, T. (1991). Partners in cognition: Human intelligence with intelligent technologies. *Educational Researcher*. 20, 3, April, 2-9.

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Technology and Problem Solving in Mathematics: Myths and Reality.

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ABSTRACT

The paper will examine the role of technological tools, especially computers, as facilitators and non-facilitators in problem solving in mathematics education. Examples of problem tasks will be given in each case. The paper will focus on over-generalizations made regarding the power of technology in mathematical problem solving. These over-generalizations (which I shall label as myths) will be illustrated by problem tasks and results of the studies that were conducted at the American University of Beirut on mathematical problem solving in schools and out-of school by students and practitioners. The possible long-term effects of technology on problem solving in non-academic contexts will be identified and discussed.

Technology is often defined as making or using of tools and artifacts. The statement attributed to Benjamin Franklin seems to imply that humans are technological animals. Are humans the only technological animals? Apes use tools with ease and skill and apes may learn the use of a tool from each other by observation and imitation. Why is it, then, that humans have advanced technologically in a dramatic way during the last ten thousand years and apes are technologically still where they were? The answer probably lies in our conception of tools and what they mean.

Tools are normally seen as those material (physical) instruments that act on objects in the external world thus changing them. (Vygotski, 1997) has introduced the concept of symbolic tools which do not act on objects but are rather psychological means of influencing the behavior of one's own or of others. Examples of symbolic tools include language, systems of counting, mnemonic techniques, mathematical symbol systems, and maps.

White (1959) hypothesizes that the ability to symbol is what distinguishes humans from sub- humans. Humans have the ability to originate and bestow meaning upon a thing or event *and* the ability to grasp and appreciate such meaning. Thus, humans have religions, arts, sciences, whereas, sub- humans do not have the ability to generate such meanings. In humans, articulate speech is the most characteristic form of expression of the ability to symbol. Because of this ability, humans can preserve their meanings in the form of symbols (language, mathematical systems, arts…), thus new generations start where previous generations have finished. Lacking this ability, each generation of sub-humans has to start anew. This probably explains why humans have progressed so much technologically while apes have not.

Human culture may be seen as the accumulation of the products of humans exercising their unique ability to symbol. White (1959) identifies four inter-connected components of human culture:

- *ideological*: beliefs, values, philosophies
- *sociological* : customs, institutions, rules and patterns of interpersonal behavior

- *sentimental*: feelings (interpersonal) and attitudes
- *technological*: use and making of tools plus ability to accumulate and progress through the symbolic faculty

Theory of Technological Determination

White (1959) advances the thesis that the technological component of culture determines the form and content of the social, philosophic, and sentimental components. Social change has historically tended to follow technological changes. In any age, the technological systems that were developed to ensure and sustain means of subsistence and protection from the elements and enemies have shaped the form and content of social organizations. The social organizations, together with the technology, determine to a large extent, the value system as well as beliefs and attitudes. It is not very difficult to see how technologies of production and defense in the last two centuries have transformed social institutions, beliefs and attitudes, as well as value systems.

Mathematics and Culture

Obviously, mathematics as a discipline belongs to the technological component of culture. As such, mathematics contributes to the determination of ideological, social, and sentimental components of culture. The common perception of mathematics as neutral with regard to culture runs contrary to the nature and development of mathematics as a product of human culture. Being purely symbolic, mathematics is one of the most powerful technologies. Mathematics as a technology derives its power from the fact that it is detached from any contextual referents and hence is applicable to a multitude of situations and contexts. In addition to impacting culture indirectly through science and technology, mathematics impacts culture (and hence, social organizations, beliefs, and ideologies) directly. It suffices to cite the tremendous impact of Euclid geometry and the positional system of numeration on the development of human culture.

Mathematics Education and culture

Mathematics education is another matter. At the school level, mathematics education may be considered as a sub-culture of the school culture that forms a sub-culture of the home culture. It is often the case that the school culture is in conflict with the out-of-school culture because of different values and technologies. Schools value the condensation of accumulated human knowledge whereas out-of school culture values immediacy, efficiency, and utility. School culture uses symbolic technologies to achieve its goals whereas out-of-school culture is heavily dependent on both symbolic and material tools.

Problem Solving and Computers

Problem solving in school mathematics is driven by two goals: The academic goal inside the school and the application goal outside the school. The academic goal in mathematical problem solving uses symbolic input and technology (mainly algorithmic) with and without the use of computers. Outside school, problem solving requires, among other things, mathematization – the ability to recognize the variables and their inter-relations which bear on the problem and at the same time to translate the problem into symbolic technologies.

The computer technology may have a tremendous impact on mathematical problem solving in academic setting. Schoenfeld (1985) identified four components of problem solving: Knowledge base, heuristics, control, and belief system. Table 1 gives examples of the impact of the computer technology on each of the four components of problem solving. The impact of the computer may be highly positive in academic settings because the former is readily accessible to symbolic input. However, the computer has a very limited role outside the school because the former does not seem to contribute to mathematization. The discrepancy in the nature of problem solving between the school and out-of-school contexts have generated a number of over-generalizations regarding the contribution of problem solving in academic contexts to problem solving in out-of-school contexts. I shall call these overgeneralizations myths and I shall use the word reality to refer to corresponding statements which I consider closer to reality and which are supported by research evidence.

Problem Solving Component	Contribution of the Computer
• Knowledge base	• Dramatic increase in the accessible knowledge base
• Heuristics	• More opportunities for effective use of heuristic: making a table, drawing a graph, …
	• Provides more effective management strategies
• Control	• Develops beliefs that are specific to the context in which computer was used
• Belief system	

Table 1: Contribution of the computer to problem solving

Myths and Realities

Myth one

Identical mathematical problem tasks will elicit the same problem solving strategies across different contexts and technologies.

Reality one

Problem-solving strategies are dependent on the context of the problem, goal and motives of the problem-solver, and the accessible tools.

Illustrative Supporting Example

Using activity theory and its methodology, Jurdak and Shahin (2000) examined the structure of the same activity (constructing solids) in school and workplace (plumbing). Data were collected from a plumber in a workshop and five high school students while constructing a cylindrical container of capacity one-liter and height of 20 cm. The actions of the students were analyzed and compared. Figure 1 presents the structure and nature of the actions of each while solving the task.

Thus, despite the fact that both the students' and plumber's actions had a common intentional aspect i.e. the construction of the task container, they significantly differed in the operational aspect (actions) or the means and concrete conditions (operations) under which such a goal is carried out. First, there is a difference in the motive of the two activities: The production of a concrete object in the course of normal job in the case of the plumber and doing a school task at the request of the teacher in the case of students. Second, the plumbing workshop and the school, in which similar tasks were learned, are two different social-cultural settings. Third, the tools that were available and accessible at the time the task was executed, were different and resulted in different actions. The tools used by the plumber were mostly concrete (hand tools, machines and basic equipment, measuring scales). The students used symbols as a "mnemonic-technical" devices and de-contextualized mediation means to calculate the unknown in the formula. Only after the second intervention, the students shifted to utilizing concrete construction tools that were available in their immediate environment. Fourth, there was an obvious difference in the constraints (operations) under which the task was executed. Whereas the plumber was constrained by the properties of the material he was working with and making the material meet the required specifications, the students were mainly constrained by translating the symbols into physical reality.

Myth Two

The use of technologies with high mathematical power in school mathematics will elicit higher-order reasoning and widen the domain of application of that technology.

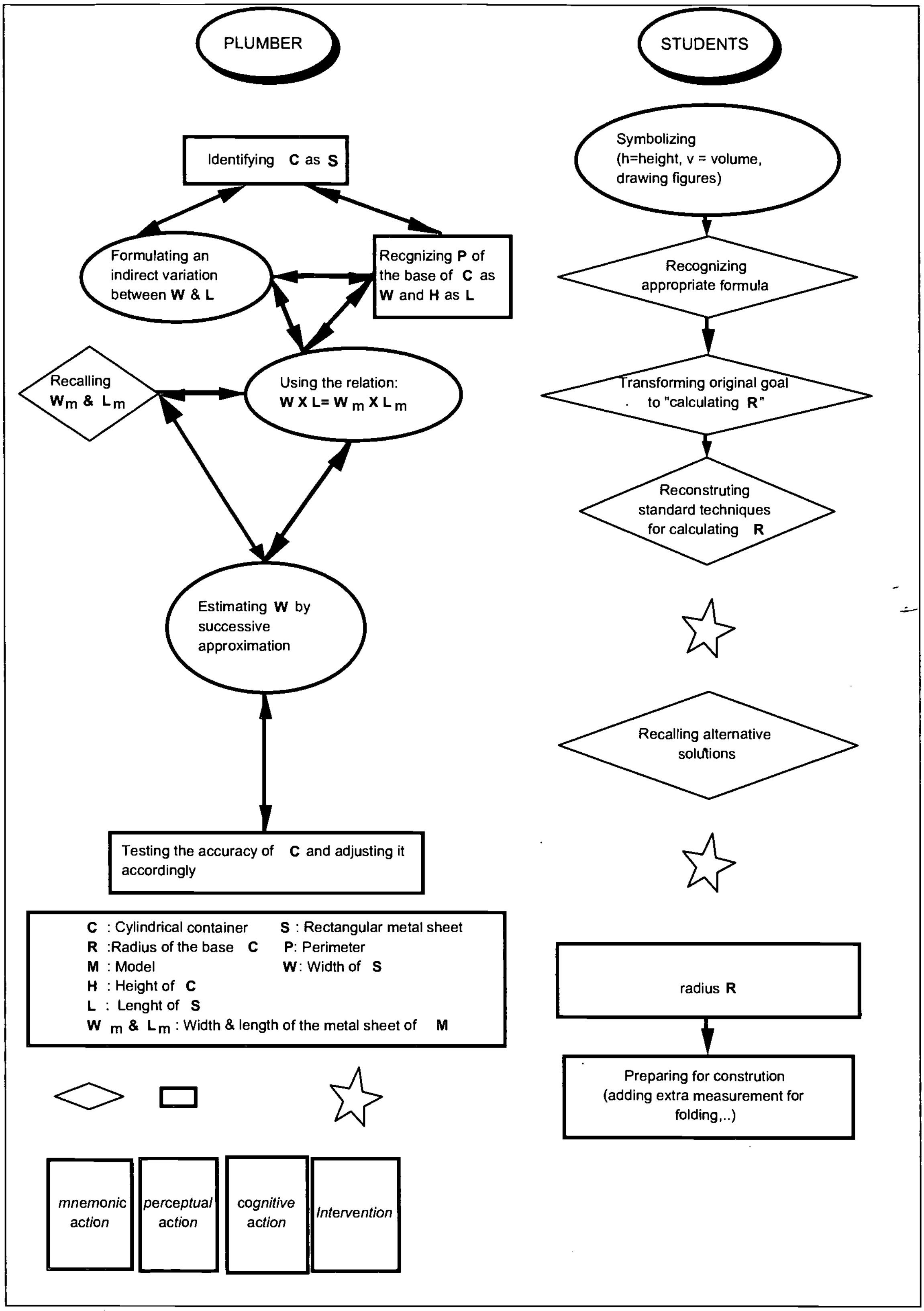

Figure 1. The structures of the internal activity of each of the plumber and the students

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Reality Two

In school mathematics, the use of technology with high mathematical power does not necessarily elicit higher order reasoning or increase the domain of its applicability in problem solving.

Illustrative Supporting Example

Jurdak and Shahin (1999) examined the computational strategies of ten young street vendors in Beirut by describing, comparing, and analyzing the computational strategies used in solving three types of problems in two settings: transactions in the workplace, word problems, and computation exercises in a school-like setting. One episode from the study is given in Figure 2. The results indicate that vendors' use of semantically-based mental computational strategies was more predominant in transactions and word problems than in computation exercises whereas written school-like computational strategies were used more frequently in computation exercises than in word problems and transactions. There was clear evidence of more effective use of logic-mathematical properties in transactions and word problems than in computation exercises. Moreover, the success rate associated with each of transactions and word problems were much higher than that associated with computation exercises.

Transaction context. *In working his way towards finding the retail price of 9 kilos of potatoes, 750 lira/kilo, Ahmed said: "1 kilo for 750 lira then 10 kilos cost 7500 lira so 9 kilos will cost 6750 lira". What Ahmed did was the following: he treated the problem that was originally 750 x 9 into 750 x (10-1) that amounted to 750 x 10 - 750 which was 7500-750 and this gave 6750.*

School context. *In subtracting 250 from 500, Ahmed proceeded mentally: "500 take away 250 there remains 250—it doesn't need calculations". But when asked to perform the subtraction algorithm using paper and pencil starting properly from right-to-left, he said: "O.K, now zero goes down, 0 take away 5 remains 5, 5 take away 2 remains 3 ". He wrote:*

$$\begin{array}{r} 500 \\ -\ \underline{250} \\ 350 \end{array}$$

Then, talking to himself, he said: "How is this? It gave 350? (Pause) ... but 5 take away 2 gives 3 (pause) 350 ? (Pause)...it is right 350". As we can see, whatever the reason, translating from mental to written form did not confirm the correctness of Ahmed's initial solution but only added to his confusion.

Source: Jurdak and Shahin (1999)

Figure2. Episode (Ahmad, 16 years old, attended school until grade 8)

Myth Three

School mathematics can be based *completely* on meaningful experiential learning using a variety of out-of-school technologies.

Reality Three

School mathematics will always include some mathematics that is not meaningful to the students at the time they learn it.

Theoretical support

There are a number of experimental studies which demonstrated the superiority of experiential learning in school mathematics. However, these studies were not based on pure experiential learning but rather on an amalgamation of learning approaches in which experiential learning was dominant. The support against this myth is rather theoretical and practical. According to White (1959), the accumulation of cultural products through symbols is unique to human culture and hence is the basis of human progress. Each human generation builds on the cultures of previous generations. Thus it is not conceivable for youngsters to reconstruct all mathematical knowledge based on experience. Moreover, it is neither practical nor economical for schools as social institutions to afford complete reliance on experiential learning

Concluding Remarks

There is an inherent conflict between mathematics and its pedagogy. In mathematics teaching, we are concerned with the *meaning* of mathematics whereas in mathematics we are concerned with its *power*. Mathematics takes its meaning from the concrete situations it refers to. On the other hand, mathematics derives its mathematical power (symbolic technological power) from being detached from the situations that give it meaning.

Since it is not realistic, and perhaps not desirable, to attach meaning to everything we teach in mathematics, it is less desirable to sacrifice meaning in teaching mathematics at the expense of its power. Hence my call for building bridges between school mathematics and everyday life including the workplace, and between the technological tools of mathematics in school and material tools outside it. Computer technology will do a disservice to problem solving in mathematics teaching if it does not build bridges with problem solving in real life.

References

Jurdak, M. & Shahin, I. (1999). An ethnographic study of the computational strategies of a group of young street vendors in Beirut. *Educational Studies in Mathematics, 40(2), 155-172*

Jurdak, M. & Shahin, I. (2000). Problem solving activity in the workplace and the school: the case of constructing solids. Unpublished manuscript.

Schoenfeld, A. (1995). *Mathematical problem solving*. New York, NY: Academic Press.

White, L. A. (1959). *Evolution of culture* .New York: McGraw-Hill

Learning with Multimedia - Mathematics Needs a Special Approach.

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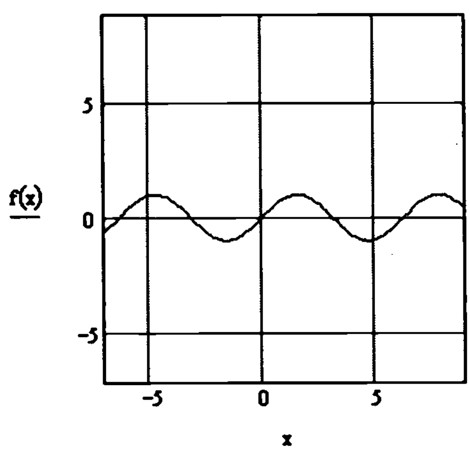
ABSTRACT

My lecture will be a sequence of examples and theoretical considerations. I start with a concrete example by which I want to show the interplay of mathematical object, medial representations, and insight. Then I will characterize typical media elements from the point of view of mathematics education: text, hypertext, pictures, videos, animations, sound, interactive settings, direct manipulations, notation systems. Some of these are then illustrated by examples from existing hypermedia learning environments. I finish with some hypertext considerations on mathematical notation systems.

Introductory example: a Weierstrass function: continuous but nowhere differentiable

The interplay of mathematical object, medial representation, and insight
I start with an example of a continuous but nowhere differentiable function, which was given by the German mathematician and one of the fathers of exact analysis Karl Weierstraß (1815-1897) and it was the first function of this kind ever given. Its definition is given by the following formula:
$f(x) := \sum_{i=0}^{\infty} \frac{\sin(101^i x)}{100^i} = \sin x + \frac{\sin 101x}{100} + \frac{\sin 10201x}{10000} + \ldots$, and its graph looks like this:



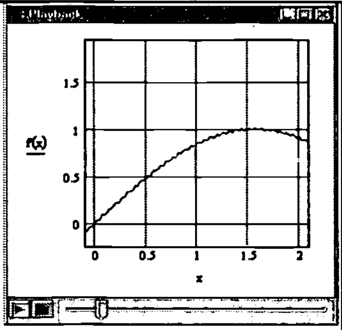
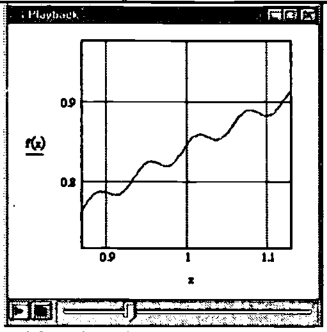
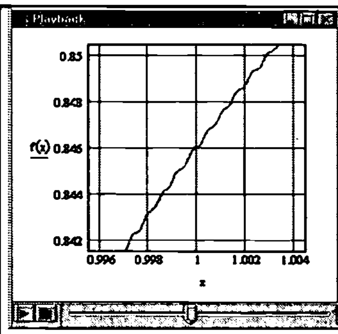
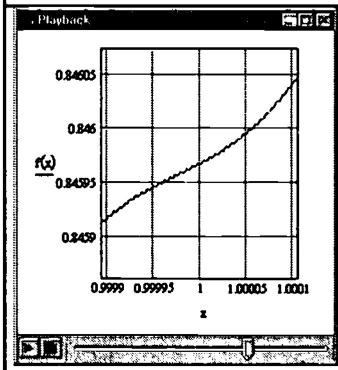
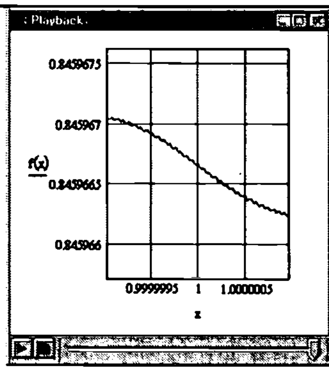
It looks like the sine function, that is the first term of the series, and indeed, the sum of the other terms is easily estimated to be at most about one hundredth, and is not visible with the chosen scale. The series is obviously uniformly convergent and is estimated as $|f(x)| \le \sum_{i=0}^{\infty} \frac{1}{100^i} = 1{,}0101\ldots$.

In the following, I will first show an animated zooming in and explain its behavior and the non-differentiability of the f, then I give some mathematical and pedagogical annotations regarding the given presentation and finish the chapter with explaining the making of the video and the necessary medial decisions.

Presentation with commentaries

I first show a small self-made video showing the zooming in on the function f at the point

[1, $f(1)$] and give some explaining commentaries afterwards by pausing at specific places of the video (the video may be downloaded from http://www.learn-line.nrw.de/angebote/neuemedien/foyer/weierstr.htm).

 Videoshot 1	 Videoshot 2	 Videoshot 3
 Videoshot 4	 Videoshot 5	A sequence of videoshots taken from the video showing the zooming in at a continuous but nowhere differentiable funktion.

In videoshot 1, the first term still dominates, but the overlay of the second term, a sine function with frequency $101/\pi$ and amplitude $1/100$, becomes visible. In videoshot 2 the first term reduces to the general direction of its tangent and the second term dominates. In videoshot 3 the first two terms reduce to a general direction (since zooming in reduces optical curvature) and the influence of the third term emerges. In videoshot 4 the 4[th] term begins to show, and at last in videoshot 5 the 5[th] term $\dfrac{\sin(101^4 x)}{100^4}$. Here the video stops, but when looking at the defining formula one can easily imagine that similar pictures would go on forever. Why does this give an argument that f is not differentiable?

Typical of a differentiable function is that it eventually looks like a straight line – its tangent – when you zoom in. This can easily be stated precisely and proved rigorously. But the video and the reflection on the infinite number of terms in the definition of f make it plausible that f will never look like a straight line in zooming in and thus f is not differentiable.

Annotations from mathematics education and media pedagogy

Where and how does insight come from?

It should be clear by now that the video by itself does not give you the insight in the non-differentiability of f. The insight comes from looking at the video and at the formula and from arguments convincing you that the video must look as it does. The visual impact must be connected to the mathematical knowledge the user already has.

As constructivism explains this is the only way to create new knowledge. Insight is always generated in your brain, media may only help in generating it.

Animation, still pictures, asynchronicity, and interactivity:

I first showed the video as a whole, but then jumped to specific places, paused there and gave specific explanations, which would have been difficult synchronous to the video running. This brings me to the role of time in the process of learning mathematics and how it is handled by different media.

Unlike a video showing real actors and real actions, mathematical videos are mostly animations which don't have an inbuilt fixed time scale. So the learner should be in control of time; he should be able to progress at his chosen pace which suits his learning processes. This important quality of a medium to allow the learner his own pace I will call asynchronicity.

Such asynchronicity is given if you read a book, but not when watching TV. In a lecture like this it can only be simulated. By looking at an animation, asynchronicity is given to the learner by the interactive possibilities of presenting videos at the computer: jumping to specific points at will, pausing, slow motion. It is even by far better realized if you give the learner interactive settings to explore instead of pre-fabricated videos, as will be shown below.

Mathematical objects and infinite processes:

Most objects of mathematical analysis are defined by infinite processes. This is true of real numbers – think of sequences of approximating decimals or shrinking intervals –, continues with derivatives, integrals, transcendental functions, or our function f. Such infinite processes can never be realized in finite settings such as computers, and in finite amounts of time. They can't even be thought of as somehow completed in real time, as the ancient philosopher Zeno has argued. But they have to be thought of as processes that can be continued as far as you wish and thereby constitute specific mathematical objects.

The understanding of such infinite processes can be immensely enhanced by considering the first steps of the real process, but necessarily leaving the rest of the process to the insight of the viewer. Exactly this I have done with the video: it showed the start of a process which you as viewers have to imagine as going on like this to infinity. Only by this infinite process the non-differentiability can be seen. This may be the deeper reason why so many mathematical educators have expressed their opinion that such monster-functions like f could not be graphically represented on a computer. Indeed, they cannot, but the start of the process showing non-differentiability can be represented, and the rest has necessarily to be created in the mind of the viewer.

Besides, this is also true of differentiability: seeing a curved function becoming straight in zooming in does not convince you of its differentiability; only by the insight that in continuing the zooming-in-process the straight line will remain straight forever, differentiability is proved.

Generality and special cases:

The Weierstrass function f is a very concrete mathematical object. But mathematics mostly does not deal with concrete objects, but deals with more general situations.

Indeed, the example Weierstrass gave was not our function f, but he considered functions of the form $\sum_{i=0}^{\infty} \frac{\sin(a^i \cdot x)}{b^i}$, $1 < b < a$. Such general objects can be described by language and algebra, but are not directly representable in graphical ways. This indicates a main difficulty in using multimedia for mathematics. One way to overcome this difficulty is using interactivity and direct manipulations like in dynamic geometry software, which was demonstrated in the lecture before by Hoyles and Noss.

CAS and interactive texts: the Studyworks file to create the video:

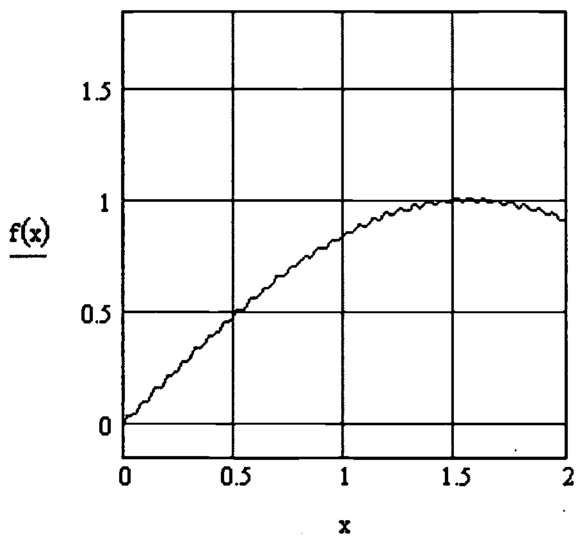
The video shown was created with Studyworks, the cheap school-version of the well-known software tool Mathcad. I show the file from which the animation was taken:

$$f(x) := \sum_{i=0}^{4} \frac{\sin\left(101^i \cdot x\right)}{100^i}$$

Hinweis

$$\delta := 2^{\frac{-FRAME}{15}} \qquad m := 1 \qquad n := f(m)$$

$$x := a, a + \frac{\delta}{100} .. b \qquad a := m - \delta \qquad b := m + \delta$$



The first thing you probably notice is the definition of the function f: the summation index only runs from 0 to 4, not to infinity. So the f as defined here is a simple finite sum of sine-functions, and as such clearly differentiable. So did I cheat you, was the video just fake?

As you can easily estimate, the difference between our Weierstrass function and the function defined here is at most about 10^{-10} in absolute value and is not visible at all within the scales of the video. So in a sense you have seen the true Weierstrass function, not fake.

But the replacement of the infinite sum by a finite one is not only feasible, it is necessary. For numerical calculations – and all plotting relies on numerical calculations – all infinite mathematical objects have to be replaced by finite approximations which can be calculated in finite – in fact short – time. For the transcendental sine functions appearing in our formula this is done automatically by the software, but the infinite sum could not be handled appropriately by the software, so it had to be done by hand. Indeed, I tried (1998) several CAS (Computer Algebra Systems): Derive, Mathcad Professional, Maple, Mathematica, Macsyma, Mathview, MuPad, TI-92, and none of them could handle the infinite sum satisfactorily, and especially no one was able to plot it.
Furthermore you notice the definitions of several variables, partly depending on the function *f* and variables defined before or on FRAME, which is the variable used for the animation. This is an interactive text as is typical of Mathcad: the variables are calculated according to their definition, and if you change the value or definition of any variable, all other variables depending on it including all plots are recalculated immediately. So the software structure behind the text guarantees the coherence of the text or at least of the connected formulas and plots. This is a feature found in many CASs using a notebook concept.
The variable δ which defines the width and height of the plot depends exponentially on FRAME which was set for the animation to run from –45 to 300. The variables m and n define the center of the plot which is taken for x from a ($= m-\delta$) to b ($= m+\delta$), the y-range extending from $n-\delta$ to $n+\delta$. For each value of FRAME, 201 values of x and the corresponding function values are calculated, plotted in the appropriate scale, and recorded for the video. If no video is being recorded, FRAME has the value 0 as in the plot shown.

<u>Interactive exploration of the general Weierstrass function</u>

This interactive text with its connections between definitions, values and plots, can be used not only for creating videos, but more directly for the interactive exploration of the setting. Thus, for example, I have set δ to 3 and changed the definition of *f* to another function of the Weierstrass function family (or an approximation of it). In a similar way I could have changed m, the x-center of the plot.

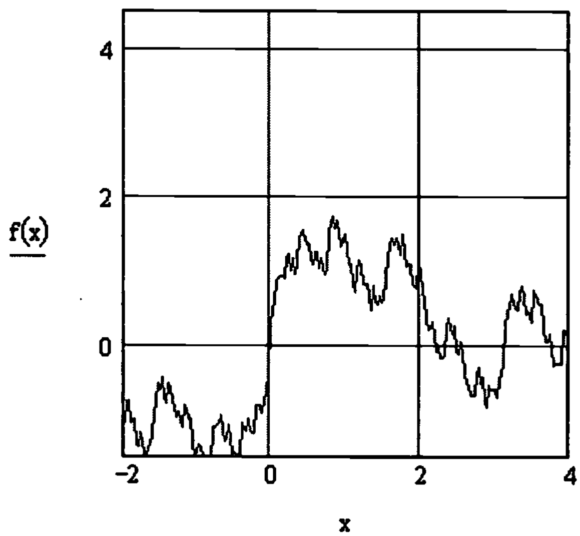
Such an interactive setting gives far more freedom to the learner to investigate, and of course this is not only freedom but also a burden. Ideally, there should be an accompanying hypertext giving the user some hints at what to look for, taking into account his mathematical maturity, knowledge, interest and available time, but designing such a hypertext is anything but easy.

$$f(x) := \sum_{i=0}^{15} \frac{\sin(2^i \cdot x)}{1.5^i}$$

Hinweis

$\delta := 3$ $\quad m := 1 \quad n := f(m)$

$x := a, a + \frac{\delta}{100} .. b \quad a := m - \delta \quad b := m + \delta$



Elements of multimedia as seen by mathematics education:

Here I am going to describe different media elements rather shortly: text and hypertext; visual elements such as pictures, graphics, animations, videos; audio elements such as sound, music, and spoken language; interactions; notation systems such as formulas and mathematical graphics.

Text, hypertext:

Mathematical texts are well-known from books and other printed materials. Books represent an advanced technology, and most mathematical texts on computers stay far behind the quality of books regarding overview, typesetting quality, navigational and search possibilities, annotation possibilities like marking or writing on margins, setting bookmarks in various physical ways, making excerpts and copying.
All of these qualities might be simulated on the computer or can be done better in principle using hypertext technology, but this requires effort and careful design. Additional benefits of computer use are the possible integration of non-static media, and of course the linking of text leading to hypertext.

Such linking need not necessarily be thought of as the rather simple linking taking place in the word wide web (www) or in Windows help systems. A mathematical text might thoroughly be linked by typed links: examples, generalizations, exercises, visualizing graphics, historical notes, real and intended applications, ..., which the learner may call for at will.

The benefits of such hypertext are:

- The promotion of autonomous learning and explorative behavior for the learner,
- The possibility of offering multi-representative views not limited by space or the line of argumentation for the author.

Thus it is possible to represent the connectedness of mathematical knowledge, which is more than just definition – theorem – proof, but consists of a wide range of connections as noted before. This representation must not be understood in the sense of so-called cognitive plausibility, that hypertext might ever be able to truly depict the mathematical knowledge in anyone's head – this knowledge is necessarily much more intricate and autosyncratic, but a hypertext may hint at the fact that mathematical knowledge has to be diverse and connected.

Of course, there are also some disadvantages: the loss of overview and orientation often noticed with readers of hypertext ("lost in hyperspace"), and the cognitive overload forced by navigational decisions, thus reducing the capacity of short term memory for the understanding of the mathematics taught by the hypertext.

Pictures and other visual elements:

Here I differentiate between pictures and videos depicting the real world on the one hand, and drawings, animations and directly manipulable visual elements of more symbolic character on the other.

Pictures and videos stress the connections of mathematics to reality: motivations through applications, introductions to mathematizations and modeling, movies on historic themes relevant to mathematics, portraits of mathematicians as in usual mathematics textbooks. I am going to show some examples of video use in the next chapter.

Far more important for mathematics and the learning of mathematics are abstract graphics, such as geometric drawings, functions plots, diagrams symbolizing sets or functions. In contrast to pictures, they can not be understood directly: their syntax and semantics have to be learned and normally are far from being trivial, as every mathematics teacher knows. (They can be described as notation systems, but this term includes e.g. number systems, algebraic formulae, set notations, too.)

Animations – which are not directly representable in static media as books – can be used successfully to represent functional relationships, parameterized objects, or processes and objects being representable by processes, as you have seen in the introductory example.

But more important than prefabricated animations are interactive settings in which the learner may manipulate the graphics, be it by direct manipulation as in Dynamic Geometry Software as Cabri, or by more algebraic manipulations: changing parameters, scaling, function definitions, and the like.

Sound and other audio elements:

As is well known, emotions and longer lasting impacts are best transported by the human auditive information channel. Thus sound is used in mathematical multimedia environments mostly for the building of emotions such as convincing of the importance of some insights, and for stressing dramatic emphasis.
But the main drawback of sound is obviously its lacking asynchronicity. Therefore the use of spoken language in mathematical multimedia is very problematic in most cases, especially if it is used to give the same information as written text on the screen.
Sometimes spoken language is used as a second information channel when a drawing is explained where the focus of the eye has to be on some aspect of the drawing, which would have been disturbed by written text. This is similar to a mathematics teacher explaining verbally some drawings on the blackboard or projected to the wall.
Of course sound plays an important role if it is the topic of some applied mathematics, as in the relationship between musical tones and trigonometric functions, or if the issue of the connection between mathematics and music is being dealt with, such as different scales: Pythagorean, well-tempered, Arabic, ...

Interactive elements:

I have already noted such elements in the description of (hyper-)texts and visual elements. Their importance is stressed by constructivist learning theories which claim that the learner has to build up his or her own knowledge by reflecting on possible outcomes of certain actions, building hypotheses, testing them, observing, resolving contradictions and testing them again.
In any case, the learner takes some actions, observes the reaction of the system/computer, and accordingly takes his next actions. This process enforces continuous active mathematical thinking, thereby hopefully deepening mathematical understanding and insight.
Many of such interactive elements have been developed in the last two decades in the creation of standard mathematical software: computer algebra systems (CAS), dynamic geometry software (DGS), function plotters, statistical packages with their linked representations, simulations of all kinds, and so on. At present we are observing the merging of these elements with classical multimedia elements as described above, leading to the emergence of new kinds of mathematical multimedia environments, but fully convincing examples and good standards are still to be waited for.

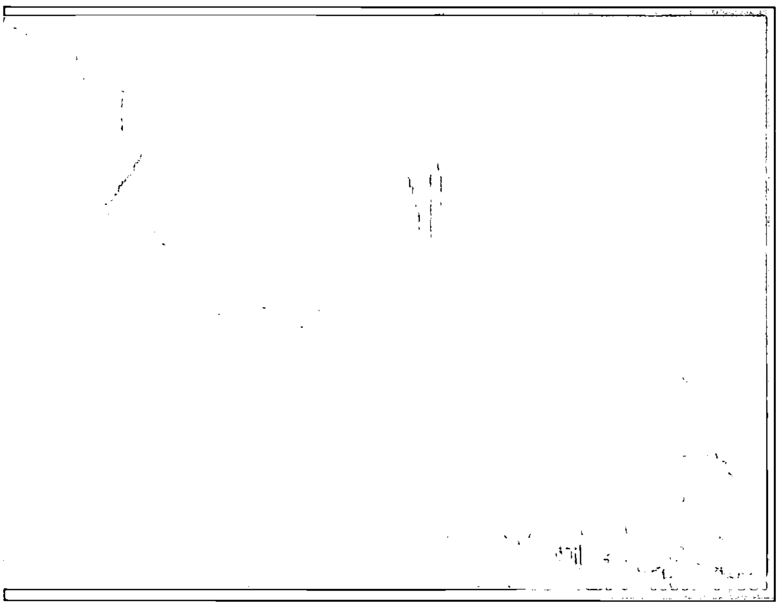
Examples of multimedia in mathematics teaching:

Here I present three examples of using multimedia in mathematics teaching: the first steps of introducing the concept of derivative from velocities in Calculus Connections, the use of reality to gain real data to prepare for modelling in Multimedia Motion, and the use of interactions to explore mathematical relationships in a piece of experimental work of German school teachers on polynomials.

Calculus Connections, A Multimedia Adventure: introduction to the derivative:

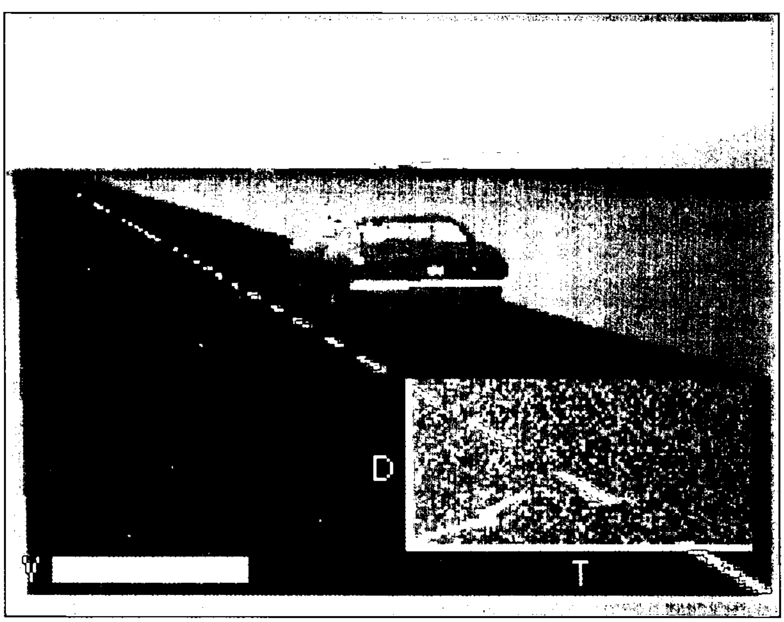
The first volume of Calculus Connections consisting of a CD and a book was published in 1995, the second in 1996, the third was announced but was never published. It did brilliant pioneering work in using videos and interactive settings to explore calculus and its connections to reality, but was poor in connecting mathematics in itself. Especially the handling of text and navigation missed all the standards I have tried to describe above.

REPRESENTING REALITY: VIDEO

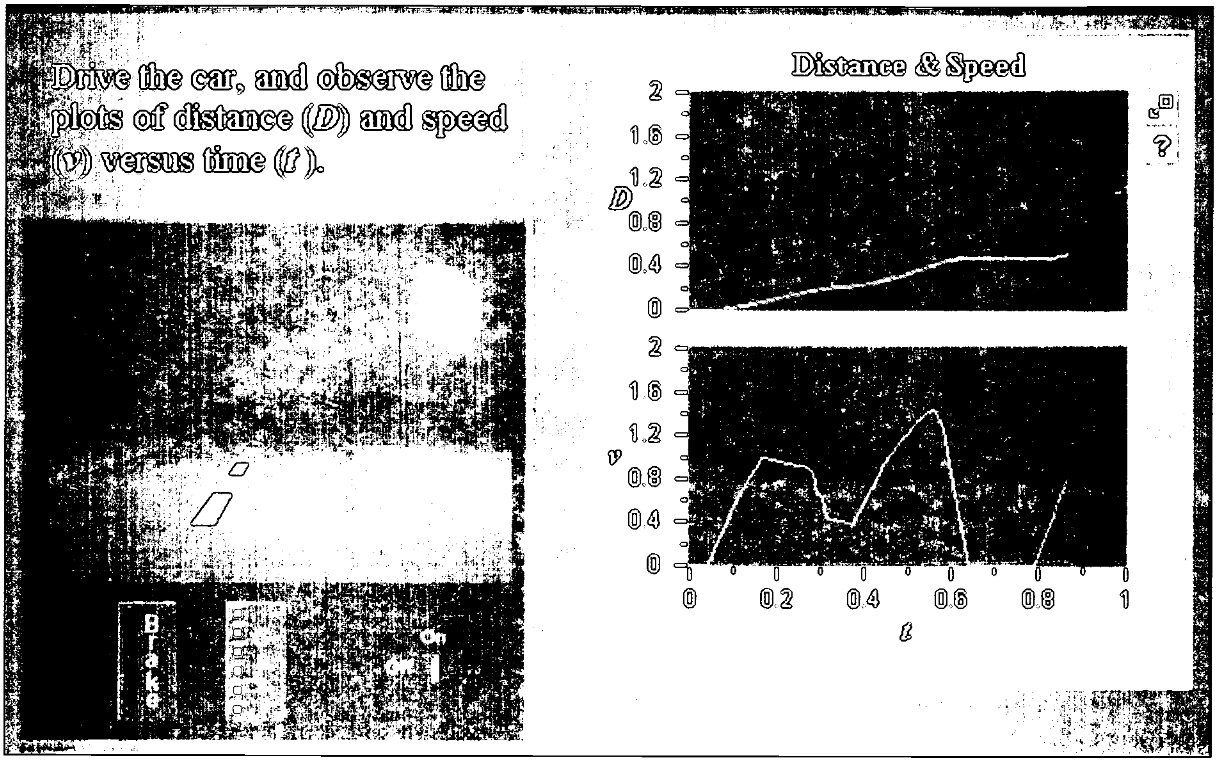


In the introduction to the chapter on the derivative a video is used showing various fast moving objects: rockets, airplanes, cars, trains. This is accompanied by spoken language explaining the importance of the concept of velocity and the need to measure it, but not using mathematical terms.

ABSTRACTING REALITY: ANIMATION

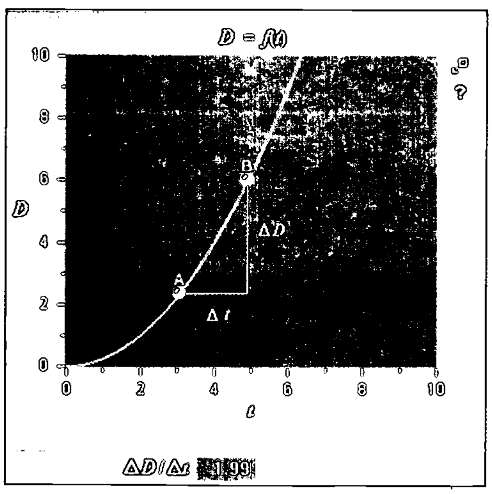


An animated drawing is used showing a car driving on a straight road with various velocities. Simultaneously a plot of distance travelled vs. time is created. Spoken language explains the connection between the magnitude of velocity and the steepness of the curve.



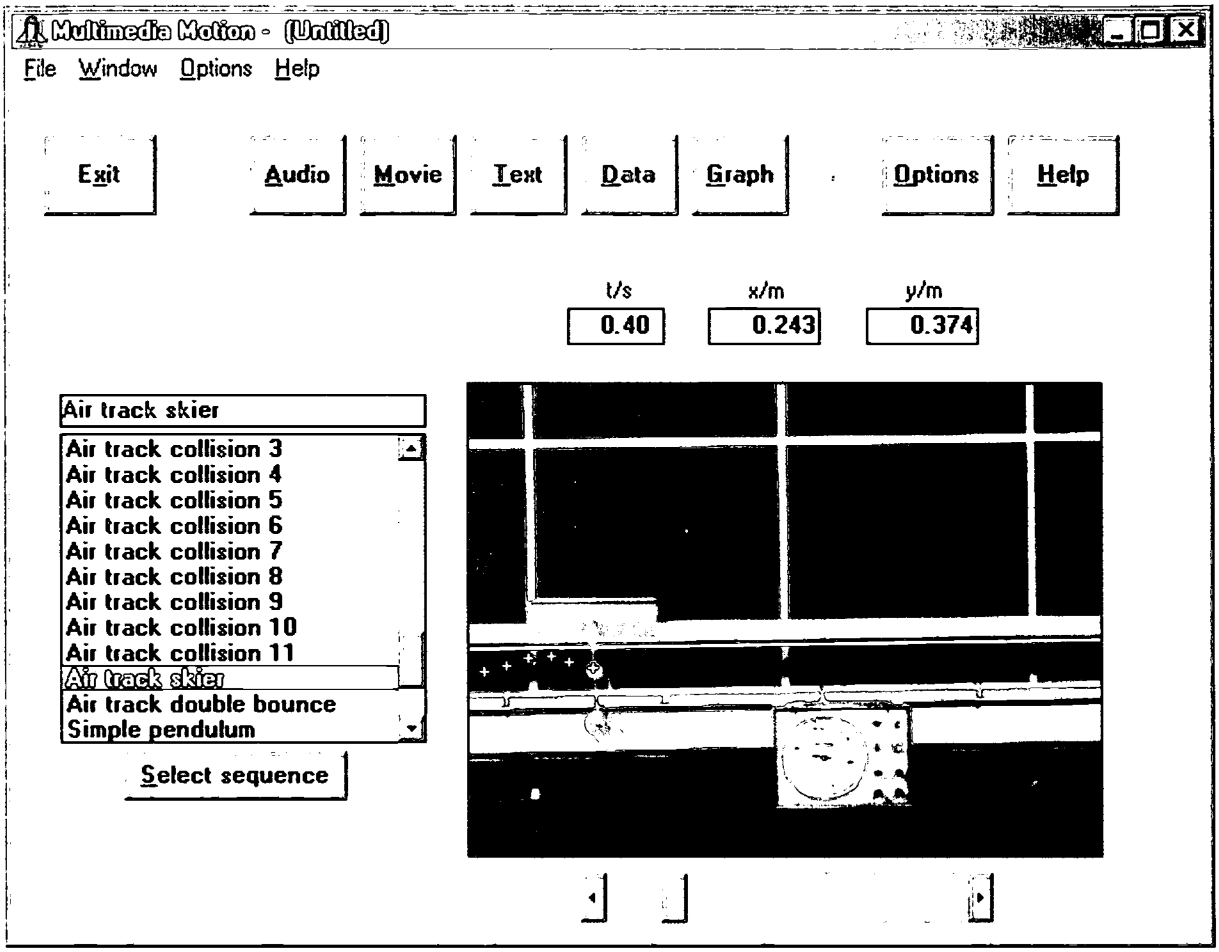
Here the user has to drive the car using the controls below the left drawing. The sound of the engine and the animation of the street markers transmit the impression of different velocities. Simultaneously the plots on the right are created according to the user's actions. The whole action takes about one minute; the dimensions of the plots are not discussed.
In the next picture – which was not shown – you can move a tangent along the distance-time-curve whose slope is displayed as a number, so it can be compared to the according value of velocity. After that the function just created is lost; there is no possibility – as in some other similar programs – to store the function or use it for further investigations.

GUIDED DISCOVERY: MANIPULATING PARAMETERS



A last screenshot from Calculus Connections: a function curve is given; the red balls A and B can be moved along the curve with the mouse at will, and the value of the difference quotient ist displayed.
The user can do only that; recording of the sequence, changing of the function or similar actions are not possible.

Multimedia Motion: observing and measuring reality



This program was made for use in physics teaching. It shows several real motions of different objects: cars, human bodies, here a pendulum fixed at a moving air track skier. They are given in carefully chosen settings so that you can do measurements on the screen. Those measurements can be recorded, analyzed and saved according to standard formats. This might be a starting point to different modelling activities.

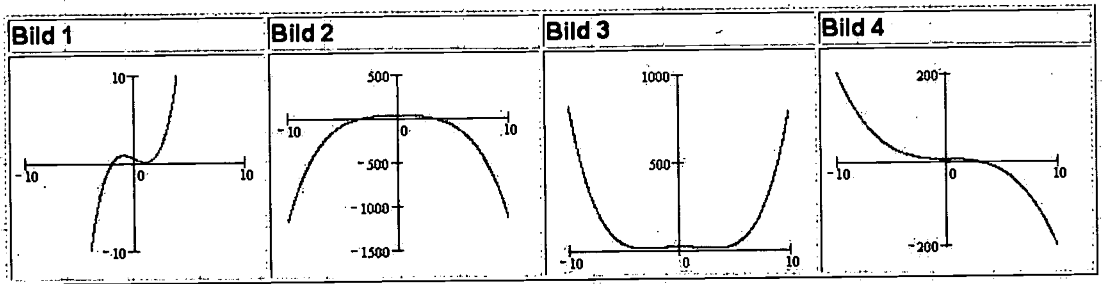
From a German teaching unit on polynomials: interactively exploring mathematical relationships

The last example [Dinslaken 00] is taken from an experimental teaching unit developed by German teachers for students in grade 11 to foster autonomous learning. It is written in html-code to be read with internet-browsers. They use the plug-in provided by Mathview, which is now called Live-Math-Maker.
The exercise starts with finding the correspondence between graphs and formulas:

Aufgabe 1

Kannst du den Funktionsgraphen die richtigen Funktionsterme zuordnen?

$f(x) = 0.1x^4 - 2x^2 + 15,$
$g(x) = -0.1x^4 - 2x^2 + 20,$
$h(x) = -0.2x^3 - 0.6x + 0.4,$
$i(x) = 0.2x^3 - 0.6x + 0.4$



The next part uses the plug-in to make the student interactively explore the behaviour of polynomials when the argument gets large in absolute value:

Declarations

$y = a x^4 + b x^3 + c x^2 + dx + e$

$y = -0.3x^4 + 0.4x^3 + 3x^2 + 0.3x - 4$ Substitute

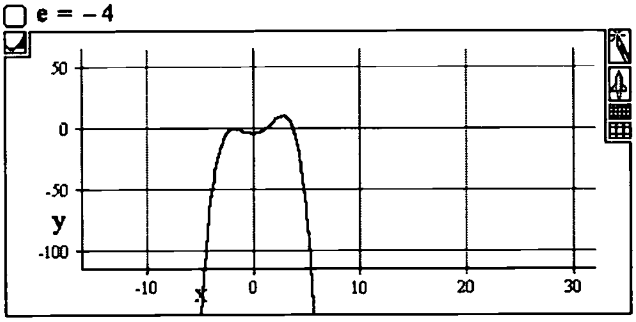
$a = -0.3$

$b = 0.4$

$c = 3$

$d = 0.3$

$e = -4$



?

Du kannst die Koeffizienten des Polynoms verändern, indem du die Zahlen markierst und sie veränderst.

Untersuche, welcher Summand des Polynoms entscheidend für den Verlauf der Funktion ist.

Versuche, eine Funktion zu erzeugen, die von links unten nach rechts oben verläuft.

Erzeuge nun eine Funktion, die von links oben kommt und später auch wieder nach rechts oben läuft.

The left part of the screenshot is an interactive text, like the one shown at the beginning of this lecture.
The German text on the right translates:

> You may change the coefficients of the polynomial by marking the figures and changing them.
> Explore which coefficient is most important for the global behaviour of the function.
> Try to generate a function running from left below to right up.
> Generate a function coming from left above and running to right above.

In addition to changing coefficients the students know how to rescale the plot using the controls in the upper right corner of the drawing.

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Notation systems and hypertext structure:

Notation systems and computers according to J. Kaput:

In describing the changing role of media settings for mathematical learning, I have found it helpful to consider the concept of notation systems introduced by J. Kaput [1992, 1994] to mark the influence of computers.
Notation systems include e.g. writing with letters (whose first occurrences on earth were found some 40 kilometers from here in Byblos/Jbeil), number systems, the system of algebraic formulae, matrices, tables, geometric drawings, coordinate systems for geometry or function plotting, etc. All these systems have in common that they have to be learned, that they are mighty tools once learned, that their power is connected to strict rules of syntax and semantics, and that their formal application may free the mind to concentrate on meanings and conclusions.
Indeed, much time of learning mathematics at school and even university is devoted to learning the operative part of mathematics, that is to learning how to use notations and how to make valid transformations within or between the most common notation systems. But of course, obeying the strict rules necessary for successful transformations exhausts the concentration capacity of many students, especially the weaker ones. Here the computer can act as support and constraint system, not allowing non-valid transformations or helping to do the right transformations. In this way, spelling checkers, CAS, DGS, interactive texts, linked representations as in statistical software can be described.
Another aspect already described by J. Kaput is the changing type of media used to embody the notations: by the use of computers, many static representations become dynamic, passive media become reactive, and display notations, that is notations only to be used to write and read, become action notations: systems which one can use to actually do transformations.
Let us shortly reflect on the last point. The clever invention of action notations in the past clearly marked many of the great break-throughs of mathematics: the replacement of the Roman number system, which could only be used to display numbers, not to calculate, by the Arabic number system, in which one could do even multiplications and divisions, was one of the technological foundations of the economic growth of Europe in late medieval times; the action notations of algebra, invented by Viéta, or of calculus, invented by Leibniz, had similar effects. Now by computers even more notation systems which were invented as display notations, have become action notations, e.g. by allowing direct manipulations in geometric drawings or function plottings or even algebraic formulae, or by allowing editing of formulae, such as in interactive texts where the software guarantees the coherence of the whole text. We are just at the beginning to understand the overall effects of these changes on mathematics, its applications, and its teaching, but they could be dramatic.

Visions of the future: mathematical notations and hypertext micro structures:

The following ideas were motivated by the latest developments in hypertext technologies where the automatic generation of hypertext links according to different user characteristics is constructively researched, and by some well-known examples where automatic links are widely used. I just mention the CD-version of the Encyclopedia Britannica, where right-clicking with the mouse on any word links you to the according description in an associated conversation dictionary. Similarly right-clicking on any object in the Windows Explorer opens up a context menu offering specific actions.

Similar context menus can be thought of with mathematical notations. For example, algebraic notations are uniformly described by the Mathematical Markup Language MathML which gives modern browsers the possibility to display them properly. But these notations can also be used as action notations: right-clicking any algebraic formula in any mathematical text could produce a context menu offering:

- transformations: simplify, expand, factor, solve for
- substitute, differentiate, integrate
- make table, plot
- other notations: input notation, structure diagram, read it aloud
- combine with some other formula
- origin of the formula (context); explain derivation

Most of these menu offerings could be generated automatically; others could easily have been generated by the author or editor of the text.

Analogous menus can be described regarding numbers, geometric figures, function graphs, mathematical concepts and phrases. The omnipresence of such offerings and links would probably be of great help and value to any learner of mathematics, indeed to most of us mathematicians, too.

Literature, links, and software

Kaput 92	James J. Kaput, Technology and Mathematics Education. In: Douglas A. Grouws (Ed.): Handbook of Research on Mathematics Teaching and Learning. Macmillan, New York 1992, 515-556
Kaput 94	James J. Kaput, The Representational Roles of Technology in Connecting Mathematics with Authentic Experience. In: Biehler / Scholz / Sträßer / Winkelmann (Eds.): Didactic of Mathematics as a Scientific Discipline. Dordrecht: Kluwer Academic Publishers 1994, 379-397.
Winkelmann 99	Bernard Winkelmann, Wie kann Multimedia das Lernen von Mathematik allgemeinbildend unterstützen? In: Kadunz / Ossimitz / Peschek / Schneider / Winkelmann (Hrsg.): Mathematische Bildung und neue Technologien. Stuttgart; Leipzig: B. G. Teubner 1999. Klagenfurter Beiträge zur Didaktik der Mathematik. S. 361-380.
Dinslaken 00	SelMa-Autorenteam der Ernst-Barlach-Gesamtschule Dinslaken, Lernen an Stationen in der Sekundarstufe II: Ganzrationale Funktionen. The URL of the example is (july 2000): http://www.learn-line.nrw.de/angebote /selma/foyer/projekte/dinslakenproj1/Station16.htm
Calculus Connections	Douglas Quinney; Robert Harding; IntelliPro, Inc.: Calculus Connections. A Multimedia Adventure. John Wiley & Sons, Inc., New York etc. Vol. 1 1995, ISBN 0-471-13794-4; Vol. 2 1996/97, ISBN 0-471-13797-9.
Multimedia Motion	The Physics of Motion on CD-ROM - Multimedia PC edition 1.1. © 1997 Cambridge Science Media. See also http://www.csmedia.demon.co.uk/
Studyworks	MathSoft: Studyworks for Schools. See http://www.mathsoft.com/studyworks/

Calculus at the Start of the New Millennium

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Background: The Rationale for Change:

Significant changes have taken place in the teaching of calculus in the US during the past decade. Since the challenges facing the teachers of calculus are similar to those facing teachers of mathematics at all levels, calculus offers a useful window on the future.

The teaching of calculus came under scrutiny in the US for several reasons. One was concern over the students' apparent lack of understanding of the subject, especially when asked to use it in an unfamiliar situation. Faculty outside mathematics frequently complained that students could not apply the concepts they had been taught. Sometimes ideas were being used in ways that were sufficiently different than in mathematics that it was not surprising that students did not make the connection. For example, the minimization of average cost is done symbolically in mathematics, if at all, whereas it is usually done graphically in economics. Similarly, line integrals and the divergence of a vector field are defined symbolically in most mathematics courses—the line integral using a parameterization and the divergence using partial derivatives. In physics and electrical engineering, however, students are expected to know from a diagram whether a line integral or divergence is positive or negative. This requires a level of visualization seldom expected in calculus courses in the middle 80s.

But students also had difficulty recognizing mathematical ideas that were presented the same way as in mathematics. A small difference in notation or the absence of familiar clues—such as "largest" or "smallest" in an optimization problem—easily threw students off. This striking difficulty in transferring knowledge between fields suggested that students' conceptual understanding was not sufficiently robust.

In addition, many students came to college believing that mathematics centers on computational techniques, rather than interpretation and understanding.[5] These students spent little energy thinking about where ideas came from or how they were used. Besides being a disappointment to faculty, these students never saw the power of mathematics to unite disparate fields.

Materials used in the middle 80s suggested that mathematicians were doing little to challenge students' views that mathematics was applying formulas. Exam questions were often of the form "Use method X to do Y"; problems in the text were usually to be done by the formula most recently presented. Consequently students got little experience in choosing a method. Even the choice of variables seldom needed thought. It was not uncommon for an entire set of exercises in a text to be written as functions in terms of x, with at most a couple that involved t or θ. Since an unfamiliar

[5]See, for example, "Are We Encouraging Our Students to Think Mathematically?" in *How to Teach Mathematics* by Steven G. Kranz, 2nd edition, American Mathematical Society, 1999.

variable is a real stumbling block to students and x is virtually never used outside mathematics, this lack of variety significantly limits students' ability to apply their mathematics.

Another concern in the middle 80s was the high failure rates in college calculus courses. At that time, the National Science Foundation projected that there would be a shortage of math/science majors in the 21st century. This prediction led to an effort to ensure that the pipeline through high school and college did not loose potential math or science majors. Calculus courses were urged to become "a pump not a filter."[6] As the first course taken by many potential math/science majors and the last course taken by many high school teachers, calculus was of pivotal importance. Transforming it would affect not only college math and science majors but also the way in which pre-college mathematics was taught.

The fact that not all students fare equally in the pipeline was also a concern: the data clearly showed that women and minorities were more likely to drop out of mathematics than their peers. Demographics trends—a much larger percentage of the workforce will be female and minority in the future—as well as simple concerns for equity suggested that the mathematics pipeline needed adjustment.

However, many mathematics faculty who became involved in rethinking calculus courses were in fact drawn in by the developments in technology. By the early 80s there were computer programs that could perform many of the skills taught in mathematics courses. An article entitled "The Disk with a College Education"[7] reflected faculty discomfort with teaching manipulations that could be done by a computer. The advent of graphing calculators in the late 1980s made rethinking the curriculum urgent.

It was fortunate that all these forces for change—concern over student understanding, the pipeline, and technology—arose simultaneously. The result was a large and diverse group of people determined to rethink calculus. The call for change fell on many receptive ears.

The Nature of the Changes That Have Been Implemented:

The changes that have taken place in the teaching of calculus over the past decade involve a greater emphasis on conceptual understanding. The "Rule of Four"—that ideas be represented graphically, numerically, verbally, and symbolically—is now widely used. Experience translating between different representations develops comprehension. Verbal explanations and mathematical modeling encourage reflection and deepen understanding.

Computers and calculators played an important role in facilitating graphical and numerical work. Thus, the modernization of the curriculum by technology also enabled an increase in conceptual understanding. Subsequent debates have pointed

[6] Calculus for New Century Conference, October 28-29, 1987.

[7] Herbert Wilf, *Notices* of the American Mathematical Society, January 1982.

out that technology can also undermine learning. However, this occurs only if technology is misused: for example, if the curriculum remains unchanged while technology is added. Consequently, significant effort has been put into identifying topics that technology makes mindless, and in investigating ways of using technology to deepen understanding.

The emphasis on modeling in many recent materials has the effect that students are now exposed to a wider variety of applications. Faculty are more apt to choose an example outside mathematics, and more likely to talk to their colleagues in the sciences about mathematics at the level of calculus. Thus, although there is still plenty of work to be done, rethinking calculus has led many mathematicians to start to build bridges to other fields. As a result of this change in attitude, the current curricular review being done by the Mathematical Association of America has for the first time requested significant formal input from other disciplines.

The changes described in this section are now quite widespread. They began in books that were specifically designed to incorporate them, but have now migrated into standard texts.

Impact of New Curricula:

The best way to get a flavor of the new course is from the problems they ask students to do. A common error made by mathematicians is to judge the level of a course by the exposition in the text. However most students at the calculus level do not read the exposition, but turn immediately to the problems. Since what students learn is determined by what he or she does, not by what he or she hears, courses should be judged by the problems they assign.[8]

Problems typical of new curricula include the following:

1. Let $P(t)$ be the population of the US in millions where t is the year. What do the following quantities or statements represent, in terms of the US population?

(a) $P(t)+5$ and $P(t+100)$

(b) $P'(1990)=2.3$

(c) $(P^{-1})'(250)=0.5$

2. The temperature outside a house during a 24-hour period is given, for $0 \le t \le 24$, by

$$F(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right)$$

where $F(t)$ is in degrees Fahrenheit and t is in hours.

(a) Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$.

(b) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?

(c) The cost of cooling the house accumulates at $0.05 per hour for each degree the outside temperature exceeds the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, of cooling the house for the 24-hour period?

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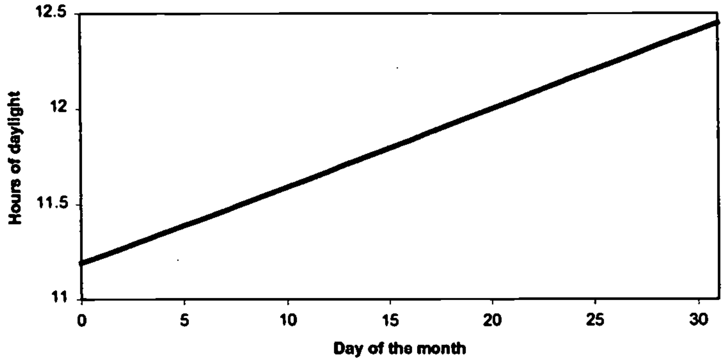
[8] This does not say that the exposition is unimportant; merely that courses should be judged by their problems.

3. The following graph represents the number of hours of daylight in Madrid for one month.
(a) Estimate the derivative, d*H*/d*t*.
(b) What is the practical interpretation of this derivative?
(c) What month does the graph show?



The first problem focuses on the meaning of a derivative, and involves no calculation. Students find this problem difficult if they do not think of a derivative as a rate of change and are not used to using units. The second problem,[9] which was on the 1998 Advanced Placement (AP) Calculus exam (a national exam taken by more than 100,000 high school students each year) required students to interpret an integral in a way that was almost certainly new to them. Ten years earlier, such a problem would not have been considered for the AP exam.

The third problem[10] also asks for an interpretation, but the difficulty here lies in part (c), which uses the fact that in the northern hemisphere there are less than 12 hours of daylight at the start of the month of March and more than 12 hours at the end. Such problems, which require information from outside mathematics, may not be appropriate in all circumstances (for example, on exams), but they do emphasize the connection between mathematics and other fields.

Although using technology in calculus is the most visible, and perhaps the most controversial, change, it is not the most fundamental. Expecting conceptual understanding on homework and exams is more important. Although faculty often spend more time designing lectures than homework, current students learn more from homework than from lectures. Thus, changes in homework and exams have a larger effect on students learning than changing lecture content. Requiring thinking is central to establishing the idea that mathematics is more than applying formulas. To the surprise of faculty, students often described the new courses as being "more theoretical" than the old. Although they are not using the word "theoretical" in the usual mathematical sense, from a student perspective they are right. The new courses require more reasoning, justification, and explanation. Just getting an answer is no longer enough.

Graphing calculators are now common at the high school level; college courses are more variable. Some decided not to allow calculators; others feel that such policies

[9] From the 1998 AP Calculus exam (slightly abbreviated).

[10] From *Calculus*, by Hughes Hallett, D, Gleason, A.M, et al., 2nd edition, p. 125 (New York: John Wiley, 1998).

undermine students' respect for mathematics and so incorporate technology. Both high schools and colleges still believe that students need skill with paper and pencil calculation. Calculators are allowed on half the AP exam; on the other half, students must demonstrate fluency by hand. Colleges often give no calculator "gateway" tests that must be passed at a high level of mastery in order to pass the course. The problems on a gateway test are likely to be routine applications of the differentiation or integration rules, such as the following:

1. $\frac{d}{d\theta}(\sin\theta^3)$
2. $\int te^t dt$

The software packages commonly used in other fields have also become more common in college mathematics. Courses for engineering may use *Maple* or *Mathematica*; courses for business students may use spreadsheets.

The use of technology has changed the emphasis in calculus courses. Students quickly adjusted to graphical reasoning; interestingly, they did not adjust as easily to numerical work. Although numerical methods were important in industry long before visualization was practicable, visualization has had the larger impact in education. Graphical reasoning has become central—graphs are now so much easier to draw that students use them voluntarily, instead of only when specifically requested to do so. This enables a wider variety of students to succeed in calculus, since many are more comfortable thinking graphically. Women students seem to particularly appreciate the increased flexibility offered by new approaches.[11]

The use of technology has generated a great deal of discussion among mathematics faculty. Much of this is a timely conversation about what we value, now that much of the computation can be automated. Some of the discussion is less fruitful, as valid concerns about preserving paper and pencil skill are overshadowed by a shrill attempt to stop the clock.

In order to remain the central discipline that it has been for the past century, mathematics will need to address two problems: it must both keep pace with the times and be able to teach thinking.

The Future: The Challenge of Computer Algebra Systems:

Although the challenges of the past decade were significant, the challenges of the next decades will be larger still. There are now inexpensive calculators that can factor, solve equations, take derivatives and integrals, and so on, all symbolically. Since symbolic manipulation is now the core of most algebra and calculus courses, changes will have to be made. Either calculators will have to be banned—a move that will not impress students—or the curriculum will have to be re-focussed. Re-focussing the curriculum involves answering some hard questions, such as:

- Which techniques are important for students to learn to hand because they generate understanding?

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[11] "Women in Consortium Calculus", in *The Mathematics Teacher*, vol 88, p. 546-549,NCTM, Oct 1995.

- Which techniques were useful in the past because they generated answers, but are no longer needed?

To get started thinking about these questions, a useful exercise is to take a list of problems, some of which should be done by hand, and some for which technology is appropriate. Faculty members then discuss where to draw the line between those that should be done by hand and those for which technology is appropriate.

There are several research questions to which answers are urgently needed. For example, it is not known about whether it is possible to teach conceptual understanding by looking at examples generated by a computer algebra system (CAS). Or is the experience of doing computations by hand crucial for developing understanding? The current faculty built their understanding through hand calculations, but that does not tell us how else this might be done. Mathematics needs a stronger tradition—similar to that in physics—of research on how students learn.

Let me give two guidelines for charting our course in the future:

- Banning calculators with a CAS will, on average, decrease students' skill with symbolic manipulation.

The reason for this counterintuitive idea is that the existence of CAS affects the dignity with which students regard the learning of symbolic manipulation. It is hard to take seriously learning a skill that can be done by machine. Thus, a course in which students know they are being trained to mimic a machine will not evoke much effort. To convince students that manipulative skills are worth acquiring requires refocusing courses so that symbolic manipulation is required even though a CAS is used.

The second guideline concerns the way in which a CAS is used. Our experience with scientific calculators shows that estimation is an essential skill, as it enables students to check their answers. The symbolic analogue is recognizing the form of an expression. This will be an essential skill for students using a CAS. Since current courses generally concentrate on teaching the manipulations, and rather than on recognizing the form of answers, the balance between the two must change:

- In courses concentrating on symbolic manipulation, more effort should be put on recognizing the form of an expression.

Notice that this is not advocating that no emphasis be put on manipulations. Just how much is needed to generate fluency and understanding is an open question.

Recommendations:

To make progress on incorporating a CAS effectively will require the collaboration of several communities. The perspective of research mathematicians and scientists is essential; so are engineers and professionals from industry. Practicing teachers are necessary to ensure that proposals can be carried out with real students. Together, these groups need to decide what is important and how to deliver it when students have easy access to a symbolic manipulator.

CONTRIBUTED PAPERS

Distance Learning Between German and Japanese School Classes Based on a Real Time Video Conference Environment.

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ABSTRACT:

We have established an environment for German-Japanese school education projects using real time interactive audio-visual distance learning between remote classrooms. In periods of 8-12 weeks, two classes are dealing with the same subject matter, exchanging materials and results via e-mail and Internet. At 3 or 4 occasions the classes met on screens interacting in explaining problems handled and solutions found. Participants were elementary school children speaking their native languages, translated by interpreters on the topics of symmetries in rectangles and stripes. It has been done with lower secondary students speaking English on topics related to discovering, proving, and applying Pythagoras Theorem. Another project was dealing with sundials and the geometry of the globe.

Background in society and education for the project:

There were two rather different observations about new objectives of education at the beginning of work on our joint project: In Japan, Yokochi was concerned with the growing demand for an improvement of the quality of mathematics education, naming intentions like originality, creativity, comprehensive uses, mathematical scholarship, etc. In this context appreciation and exloitation of different cultural characteristics as well as interdisciplinary components (mathematics and arts) played highly influential roles.

In Germany, reflecting activities for more general intentions like interdisciplinary and intercultural context within subject matter education, Graf had pointed out that decision makers in education agree that modern societies rely increasingly on larger numbers of individuals with high-level knowledge and skills at their disposal (Graf 1995, referring to Ruiz, 1993). This means, in addition to elementary cultural techniques such as reading, writing and calculating, school has to teach other skills such as analysing, abstracting and modelling. Conceptual and communication skills are also required, in the fields of production and service, e.g. More, an increasing range of jobs depends on individuals who know about their responsibilities in a wide context, and many of these people must be able to work in multi-disciplinary teams.

When exchanging these observations, we agreed that in Japan and in Germany as well as in other countries there are challenges to societies by universal problems such as the world economy or environment. These problems are of extreme complexity and require immense international interaction. Any success is dependent on mutual understanding and acknowledgement of different traditions and attitudes related with problems like mass production, environmental protection, political and social structures. Readiness and goodwill have to be developed in young people growing into these societies, so more attention has to be given to interdisciplinary and intercultural intentions in education. There are avenues in many subjects in school to pursue intentions such as the ones discussed above and every school subject could contribute. In particular, information and communication technologies form a rich source for contributing to such mutual understanding, linked with subjects like mathematics or social science e.g., carrying the high-level knowledge and skills mentioned above.

Basic structure of the learning and teaching experiments

Several distance learning experiments of Yokochi's research group, including pairs of classes from distant provinces in Japan had shown that mathematics education as well as science and art education could profit considerably from theses activities. This encouraged us to plan practical work growing out of the very general considerations extended above. In a series of distance learning experiments between school classes in Japan and Germany, it could be demonstrated that these activities form an excellent platform to promote intentions like the ones mentioned above in school education. Each experiment consisted of different activities of two partner school classes and their teachers, together with researchers in didactics at universities, extending over six to twelve weeks. The activities started with preparation of a project by researchers and teachers, contacts running via mutual visits in Germany and Japan or via email, Internet homepages or air mail. After this, teachers in the two countries started working with their classes concurrently on the same topic, mostly taken from mathematics, social science (environmental problems) or arts.

Besides learning about the topic in different ways the students, in more or less co-operation with their teachers, started to prepare demonstrations for their peers about their findings and results. These were executed in several video-conferences, integrated in the total activity of 6 – 12 weeks. Material was also forwarded by email or fax or put in the respective homepages in the Internet. Some conferences consisted in the teaching of the teachers about the mathematical background of some products generated (tiles with patterns e.g.), sometimes groups of students did the teaching (see (5): methods).

Beside concentration on special subject matter – which was essential in our design of the learning units – there were passages in the activities, per email or in the video conferences, where the students tried to find out more personal information about each other concerning pets, grading systems, views on the other countries or people, including the students themselves.

Findings for general and subject matter education through the project:

First let us mention that our experiments did not only effect the learning and understanding of the students. At the same time, the researchers from teacher education as well as the teachers active in the preparation, execution and evaluation of the distance learning experiments profited a lot. They had to reflect deeply on the educational theories they were relying on, so that they could be understood by their partners in the other country. Very much feedback had to be given to make sure that the others' intentions had been understood. It became clear that understanding went far beyond the language problems. Still, using English to explain activities based on German or Japanese language was a problem in itself. The teachers had extra work to invest since they had to explain their project in their environment, i.e. to colleagues, school administrators and parents. They experienced that both sides were very ambitious to "perform" good presentations and they had to control these tendencies for the sake of allowing genuine learning processes.

Before we give detailed descriptions of students, contents, methods, media and other topics determining our learning units, we want to list some general findings.

(d) The students' interest in each project was great and holding on for the whole period of the learning units, and after

(e) The students were planning with great zeal how to explain their results to the pupils in the other country

(f) They were very interested in learning and understanding what they were told by the other students and how those solved their problems

(g) They picked up their ideas and adapted them when doing their own work

(h) They developed a very general interest in the other classes environment and country, also in individual peers

(i) They discussed the project with other students and teachers from their schools, and with their parents and families

(j) In the sessions they were extremely attentive and ready to perform

(k) They were competing in a friendly way; when they put questions they reinforced good answers by cheering.

(l) Without any problems in using the new technology they put direct questions to their peers in the other countries, forgetting about the distance

(m) They patiently waited for translations or – when English was the joint language of communication – for repetitions of statements and questions

We also had positive findings about special intentions and goals related to the subject matter chosen. In most cases it was geometry. Learning progress in some specific matter was always the main intention of the experiments. We took great care that neither just general intentions like discussed above dominated the activities nor mere use of technology.

Especially for mathematics education we had put up a list with aims for the reorganization of this subject:

(n) Initiating mathematical originality and creativity of students

(o) Learning comprehensive uses of mathematics with other subjects through solving real problems

(p) Improving the mathematical scholarship of students

(q) Appreciating and acknowledgement of the mathematical cultural characteristics of each district or country in problem solving

(r) Cultural exchanges related to science and technique between students of two classes

(s) Interaction with peers from different regions in mathematical problem solving.

Interviews with the students as well as their behavior and performance have shown that many elements of these purposes were attained through the intercultural learning units of our experiments. This confirms with the results reported by Yokochi and his project group about internal Japanese distance learning activities. Students took up contents as well as methods from their partners and used them when solving problems or creating art objects, e.g.

Age levels and contents of the experiments:

The following partnerships and learning units between German and Japanese classes have been realized so far:

(d) Peter-Witte-Primary-School in Berlin and Primary School attached to Yamanashi University in Kofu. The students came from grades 5 and 6, there were 20 in Berlin and 40 in Japan. Their subject matter was stripe patterns and rectangular patterns, seen under aspects of mathematics and arts. There were four video conferences of one hour each included, two dominated by students presentations and discussions, two by teaching of the teachers to remote classes. Two German teachers performed team-teaching for some periods.

(e) Hildegard-Wegscheider-Secondary-School in Berlin and Irihirose High School in Niigata. The students came from grade 10 in Berlin and grades 8 and 9 in Japan. Subject matter was the Theorem of Pythagoras. Groups of German students explained proofs on different levels: experimental work, visualization by graphics and logical proofing based on constructions at the blackboard. This took about one hour.

(f) Hildegard-Wegscheider-Secondary-School in Berlin and Secondary School attached to Yamagata University in Yamagata. The students came from grade 10 in Berlin and grade 9 in Japan. The subject matter was the sun-dial and its mathematical background from the geometry of the globe. Like in b) this time groups of Japanese students taught the German students using models and logical explanations at the blackboard. They put excellent questions to the German students and they showed great excitement on good answers.

(g) An experiment at university level was performed between a research lab of Mitsubishi Electric and Freie Universitaet Berlin. German Students from Mathematics, Computer Science and Physics were instructed by researcher Dr. Toshio Ito about computer simulation when designing kites, including video simulation using PC. The conference was interactive so that students could put questions.

Three more projects were dealing with presentations of Tokyo and of Berlin in Japanese language, with 'cleaning of the classroom' by students and with 'saving energy in school'.

Teaching and learning methods at the experiments, including language problems and solutions.

Different methods were applied and experienced. In the Primary School experiment both sides followed a rather conservative mode of teachers on both sides controlling the performance of the classes, putting most of the questions etc. The pupils were given opportunities to present their works by prepared statements. Students and teachers spoke in their mothers' languages, they were translated consecutively by interpreters in both classrooms.

The Pythagoras experiment was characterized by independent teaching of German groups of students to the other side. They had explored the topic in the Internet and evaluated the results together with their teacher. The effect of learning was considerable on both sides, since there was a high motivation to good teaching and understanding. This experiment was also characterized by both classes speaking English. This caused many misunderstandings and repetitions. Using figures and visualizations was a relief in this situation.

The sun-dial experiment was also conducted in English with the Japanese students doing most of the teaching in small groups. This was an outstanding event in a Japanese classroom.

We conclude this section with a general remark about the role of teachers in our special learning setting, which could have some relevance in general. We started our experiments with teachers organizing most of the planning and the execution of work in the classes. The students' roles were mostly to demonstrate what they had learned. Elder students then took over some of the teaching; this was still organized or "filtered" by the teachers. With growing experience, the students' demand grew to erase the filters totally, including the activities of preparing to teach and so to replace their own learning by preparing to teach. This, of course, causes a gain in active learning, but at the same time a loss in efficient learning.

School environment and media.

Experiments as described can only be performed in a very friendly environment. It certainly became clear from the descriptions above that many questions about the "how" and "why" of such experiments will arise. To accept the answers, decision makers in education, responsible for the schools, teachers, parents, and sponsors have to be very far-sighted and open-minded.

Students and teachers will have to invest very much time for such projects, and they have to be very patient. Intercultural understanding and co-operation is very rewarding – but it is hard work at the same time. Language is one problem; different thinking and planning styles are others.

Cost for equipment is considerable and so is the telecommunication. The minimal equipment is a PC, a beamer plus a standard teleconferencing unit (about 2,000 $US). If PC and beamer are not available, a total of $US 10,000 will do. Communication needs 2 ISDN lines carrying 64 kilobytes per second each. Considering the development in telephone cost, a price of one dollar per minute can be expected, even between continents.

The equipment that we used in Germany and Japan was sponsored by Mitsubishi Electric. The company had expanded the structure of a teleconferencing system considerably to meet the demands from two school classes "conferring" (Koizumi and others, 1999). So the systems used a 42-inch monitor in the German classroom (about 20 people) or a LC-projector on an 80-inch screen in a Japanese classroom (about 40 people).

There was a panorama camera and a motorized camera zooming and following individual persons or groups in each room. Video recorders recorded incoming and outgoing picture and sound for later evaluation. A camera projector (stand camera) was used to transport any pictures and graphics to the remote monitors. Besides controlling the system, the PC could be used to create and transport computer screen images. For good sound transport a complicated system of ceiling microphones was installed. Besides a basic readiness to use such equipment for distance learning as indicated above the system demands technical and didactical or methodological knowledge about the best use from teachers and students. This implies in-service teacher training and better pre-service teacher education.

Conclusions and visions.

Neither the cost of the equipment and communication nor the extra amount of work related to the new media for distance learning should discourage us from integrating these media into the teaching and learning processes. This situation is comparable to the days when PCs first entered some schools and appeared un-payable and unusable for education.

Teleconferencing systems or what will be developed from them will be a standard medium in schools within several years. They will not only be used for communication with peers, which we consider a very fruitful application, but also for communication with any point on the globe and anybody on the globe who can contribute to the demands of students to get information in an interactive way. Teachers will have to become advisers and supervisors for this learning by exploring. They have to prevent students from wasting too much time by unqualified exploring.

Distance learning originated from the problem of physical distance in time and/or space between a class and a teacher, or, more general, between learners and knowledge bases including intelligence to interact – real or artificial. The more this intelligence and its means to interact (through multimedia) grow, the less the physical distance will matter. Instead, cultural distance as we used it in our experiments will become important for the learning process between learners and an intelligent source of knowledge. This distance can cause the learners to get more provoked to learn from a distant source, be it a group of peers or a knowledge base, to learn more through competing, or just through imitation and variation.

Putting individuals into school classes for learning may have economical reasons. But – each student coming from a different environment (the "culture of a family", e.g.) – it also means to initiate some kind of distance learning. This idea can be forwarded to the family of man.

References:

Downes, Toni and Zammit, Katina (2000) New literacies for connected learning in global classrooms, in Taylor, H. and Hogenbirk, P. (Eds.) *The Bookmark of the School of the Future.* Boston/Dordrecht) London: Kluwer Academic Publishers.

Graf, K-D (1995) Promoting interdisciplinary and intercultural intentions through the history of informatics, in D. Watson and D. Tinsley (eds.) *Integrating Information Technology into Education.* London: Chapman & Hall, 139-150.

Graf, K-D and Yokochi, K. (1998) Educational experiments of distance learning and reorganisation of mathematics education, in D. Tinsley and D. C. Johnson (eds.) *Information and Communication Technologies in School Mathematics.* London: Chapman & Hall, 265-274.

Koizumi, K., Dasai, T.S., Moriya, S. [1997] Proposal of interactive distance learning with CCV educational system and verification experiments. In *Proceedings of the Int. Conference for Computer Communications* (ICCC'97), 107-114,

Koizumi, H., Dasai, T., Yokochi, K., Graf, K-D, and Moriya, S. (1999) An interactive distance learning system and verification experiments between Japan and Germany, in D. Watson and T. Downes (eds.) *Communications and Networking in Education.* Boston/ Dordrecht/ London: Kluwer Academic Publishers, 39-50

Ruiz i Tarrago, F. R. (1993) Integration of information technology into secondary education: Main issues and perspectives, in Van Weert (ed.) *Guidelines for Good Practice, IFIP Working Group* 3.1, Geneva.

Wibe, J. and others [1999] Projects in Networked Learning: European experiences, in Watson, D. and Downes, T. [eds] *Communications and Networking in Education.* Boston/ Dordrecht/ London: Kluwer Academic Publishers, 101-126

Attitudes and Concerns on Distant Learning in Lebanon: A Multiple-Case Study*

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ABSTRACT

This study investigates the worth and value of distance education in Lebanon. The study surveys 7 school administrators/policy makers and 112 school teachers equally divided among fourteen urban and rural areas. Attitudes of administrators/policy makers revealed a degree of negative assessment toward the workability and training needs of teachers. Administrators/policy makers were more concerned with the situational and financial factors, teachers on the other hand felt that training was a necessary goal to familiarize with new technologies. School administrators felt that costly training and the purchase of technologies was inconceivable, more than 50% of the teacher sample reported little acquaintance with distance education, a degree and exceptional high level of unawareness recognizes the need to organize workshops and technology seminars for which private schools can undergo structural, curricular and pedagogical practices needed for a fully blown distance education program in the Middle East.

Introduction

The most important movement of the 21st century draws on the role of education as a tool for responding to a challenging and exponentially growing information economy. As there is widespread skepticism as to the ability of educational systems in developing countries to accommodate for the challenges of the information expediency, developing Asian countries have invested heavily in education and currently provide a large part of higher education by distance (Tam, 1999). A negative view of distance education, however, downplays the role of educational transmission among academicians and policy makers in developing countries. As there are no certification for distance education degrees in many Middle Eastern countries—partly because it undermines traditional education and limits students' interaction with peers and faculty— it does away with the platform for which a deliberate academic discourse within an academic community takes place (Mathews, 1999). For example, many Middle Eastern countries (e.g., Arab Gulf and Lebanon) heedlessly give way to branching out education to the remote areas through the construction of university campuses often patterning after traditional Western university models. To under estimate the value of distance education, as the next potential system for higher education in the Middle East is a great loss to further symmetrical development and interactions that bridge the existing societal, economic, and political gaps among Middle Eastern societies. By contrast, many nations have stridden to accommodate for larger audience through distance education. Universities in the US, Asia, and Europe, for instance, explore the possibility that in the year 2001 more than 75% of the colleges and universities will use distance-learning technologies as the main medium for the cultivation of global knowledge (Radford, 1997).

If we consider the globalization of modern schooling based on the work of Boli and Ramirez (1986) education is seen as a social institution, a transnational or "world cultural" phenomenon where educational developments evolve at the level of world cultures. This conception is argued for by an information driven society which demands other means than a traditional medium of instruction. Certainly, it is envisioned that learning will be more global oriented rather than particularistic. The fact that distance education overcomes the tyrants of distance, students will no more be bulwarked by the factor of distance in selecting a university system.

Alas, in many Middle Eastern countries there has been no real implementation of distance education in higher education. Strategic thinking regarding distance education is counter weighted by a negative assessment. Neither policy nor basic research has taken any form to explore the feasibility of distance education in the Middle East. However, recently there have been attempts by local institutions and international organizations like UNESCO to promote regional cooperation through distance education. However, despite these attempts there are certain difficulties for their implementation such as the beliefs that militate against distance education as an instrument for globalizing knowledge.

While policy may be at hand, researchers warn of an inherent problem associated with ignoring classroom teachers' beliefs about any form of implementation and use of the technologies for distance education (Czerniak, Lumpe, Haney, and Beck, 1999). The "top-down" approach draws some worry from researchers in the field. If there is such a belief among policy makers concerning the implementation of distance education programs, it is fully an initiative of an administrative and funding body; it lacks the organizational support system it needs for its success by those who will run the program. For instance, Bybee (1993) suggested that restructuring science education classroom teachers could determine whether these programs succeed or fail. Schuttlofel (1998) also argues that technical capabilities of technology as well as the belief systems of teacher are major determinants to a satellite-compressed video as a medium for distance education. Not only teachers' knowledge or skills are needed for implementing distance education, but also their perceptions and attitudes of the use of technologies for the purpose of education (Tobin, Tippins, and Gallard, 1994). The needs to reform existing educational programs and implement new ones require the understanding of teachers' beliefs concerning distance education. It is assumed that those who may be involved in distance education programs consider that literacy, awareness, and interest as powerful agents for curricular change and alternative means to the traditional method of instruction.

Several requirements for the successful implementation of distance education are required. These are: (i) preparing material; (ii) managing the dissemination and exchange of information through technological means. Are teachers and administrators prepared for undertaking the above functional requisites of distance education? Will they be able to propagate or hinder a program of distance education whether through management or use of technology? This research is controversial because it does not only assess the attitudes to the worth of distance education among school administrators, policy makers, and teachers but also because it shows a *decalage* within school systems. In particular, between the traditional and more pragmatic patterns of education and between the attitudes of teachers and administrators as well. Taking account of contextual factors the rationale of this paper is twofold: (i) teachers' believes are more effective in inducing change (McLaughlin, 1990); (ii) teachers' believes are a clear "guiding sticks" in the planning procedure for distance education and classroom practice (Tobin, Tippins, and Gallard, 1994).

The Conceptual Framework:

This research is based O'Malley and McCraw (1999) work which adapts Roger's (1995) diffusion model. The model suggests that for any change in the form of innovation and decision-making process, five stages should be accounted for Knowledge, Persuasion, Decision, Implementation, and Confirmation. The fact that telecommunication and distance education have not found their way in the remotest areas of the Middle East and particular in many parts in Lebanon, they remain as pursuant force in the expected development for change in society. Hence, change is still at the stages of knowledge and persuasion. In this study, we adopt two of O'Mally et.al. three constructs: (i) prior educational conditions; (ii) perceived characteristics of distance education. These two provide the framework of teachers' attitudes as indicators for the advantage, the success, and feasibility for executing a distance education program.

Objectives:

The purpose of the study was to gather information about teachers and school administration perceptions of the value of distance education. While expected perceptions were considered to be neutral based on the premise that teachers have little acquaintance with distance education and because telecommunication services are meager in rural areas, these mirror the same picture in other developing countries (Van Koert, 2000). Of the various computer-mediated instructions only, the Internet is the accepted medium that many may, at personal level, register in a program or in an institute of higher education that provides instruction in an asynchronous manner. Owing to the lack of information access and to the fact that the Internet is the only accepted form of communication through distance, assessing attitudes of distance education provides a general functional definition to teachers as conceptualized and annotated by Keegan (1980). The definition is presented in the questionnaire in the Appendix.

Our basic premise starts with the idea that teachers are novices in their use of technology. Prior to the survey of teachers' attitudes towards the value of distance education, the authors conducted an assessment of school administrators' views of distance education programs, training needs and the readiness of schools to participate in distance education. A general perceived worth would provide the reader with those dimensions that may indicate the relative advantage of distance education over other traditional methods of education.

Methods:

In order to assess teachers' training needs and portray their views regarding distance education, open ended interviewing technique and main attitude scale were the main tools for data collection (Cohen and Manion, 1994; Singh, 1993). The interviewing schedule was laid out in such a way as to generate information by a flexible approach. The interviews conducted with school directors and teachers centered on teachers' training needs in light of the tectonic shifting in both styles and content of pedagogy proposed by the New Framework for Education in Lebanon. Additionally, the core educational issue to be explored by the researcher has been the very idea of distance education. Pre-pilot runs and consecutive informal panels were tried with students and university faculty to establish the validity of the interview schedule and ensure its usefulness in gathering of relevant information.

The fact that data collection only operates as indicator rather than confirmation, this led us to require cross-checking of the data through the means of triangulation. Thus, in addition to the interview schedule, a survey questionnaire was constructed by the authors, which sought to gather information concerning teachers' views of the value and worth of distance education.

The questionnaire was adapted from Bratina and Templeton (1997) and O'Malley and et al., we constructed a 17 item, 7-point Likert-scale ranging from strongly disagree to strongly agree. The main dimensions focused on the worth, feasibility, application, training, practicality, and educational advantages of distance education. Items that were judged as attributive to construct, ratings were summed and divided by the number of items obtaining a mean rating on a specific construct. For instance, those items that reflected worth of distance education were summed and divided by the number of items that reflected the specific construct. To overcome the within and between confoundment, the mean rating was obtained for specific items reflecting certain constructs. Then the neutral point was incorporated in which a t-test was used to determine whether the ratings departed significantly from the neutral point.

The instrument was administered among teachers as potential aspirants for distance education. The teachers were given a functionary definition of distance education and directions were given to rate the 17-item scale. The responses of the questions were data recorded, and analyzed. The semantic differential items presented direct measures of attitude toward distance education feasibility, application, practicality, and educational advantages. Validity of the instrument can be inferred by its adaptation to criterions. A panel of judges and administrators consensually assessed the applicability, adequacy, and appropriateness of the instrument. Convergence among judges provided a positive assessment based on objective criteria.

Sample:

Eleven private schools situated in urban areas were opportunistically selected for the study. In order to allow for a rural-urban dimension in the analyses of data, four private schools situated about and from the North of the country were also selected. Of the eleven schools, two had complete educational programs from Kindergarten to Baccalaureate II (13th grade), one had a Kindergarten program only and the fourth (rural) had elementary and intermediate cycles only. The media for instruction employed in two urban schools were English, French and Arabic, whilst the kindergarten school teachers communicated in both English and French. Rural schools were bilingual i.e., English and Arabic. From these schools a total of 112 teachers were surveyed. In addition, seven directors drawn from seven private schools were selected for the study. The measure on the scale was further supplemented and verified with the interview data.

Results and Discussion:

Interview data with school directors and teachers raised a number of issues, which inadvertently went beyond the boundaries of the study. These issues are worth reporting having broader relations with existing limitations connected with implementing distance education in schools. There were several specific areas on which the researchers focused: worth, effectiveness, feasibility, application, practicality, educational advantages, and training needs.

In a hierarchy of satisfaction of basic needs where day-to-day-life in Lebanon is demanding, and in light of the limited professional rewards from teaching, judging by what school directors reported, the furtherance of education through distance was not teachers' highest priority. Teachers, on the other hand, were neutral about the benefits of enrolling in a distance education program, i.e., neither did they feel that distance education had any worth or contribution to their training needs nor any benefit for their professional development (t=0.35, df=111, p>0.05). This finding concurred with Yelland and Bigum (1995) who suggested that teachers' horizontal development may be the more appropriate practice and could maintain teachers' networks that give purpose, focus and support. In comparing the mean rating of teachers on the effectiveness of distance education, we obtained a t=-0.33, df=111, p>0.05 by aggregating mean of items 1, 2, 13 and comparing the middle point (4) on the scale. Teachers were neutral about giving time out of their teaching hours for preparation for higher education through distance (t=0.28, df=111, p>0.05).

The study evinces that there are cycles of commutative ignorance of the idea and utility of distance education. Interviews results show that there is wariness (if not outright rejection) among schools regarding distance education. The researchers during interviews conducted with one school director who said that the idea was new to her and even strange that one could suggest it as an alternative model to education. Teachers' attitudes towards contributing to a distance education program were more positive than those expressed by administrators. Attitudes were different among those who were acquainted with the nature of distance education and those who were not. We ran a one-way ANOVA with two groups, those who were acquainted with distance education and those who were not on the rating of item 17. A significant difference F(1, 105)= 4.088, p<0.05 was found for teachers who were acquainted with distance education by rating higher (M=4.83, SD= 1.27) than those who were not acquainted on their ability to contribute to a distance education program (M=4.23, SD =1.68). Lack of familiarity with distance education or lack of understanding the strategies for using the technology for distance education are major problem areas for those who are advancing a program in distance education (Sherry and Morse, 1995).

In a further exploration of the possibility for introducing a distance education program in schools in Lebanon, a school director established several impasses to the idea. He described it as a very costly one, which would require the recruitment of additional teachers into each of the departments. He added that the school might not even be capable of securing salaries for more teachers needed for running the program. Teachers, by contrast, disagreed to a lack of money, trained people or workability necessary to run a distance education program was less than average (M=3.98, SD=1.58) and insignificantly different than middle point 4 (t=-0.013, df=111, $p>0.05$). In asking a director of rural school about the difficulties faced by his own school, he preferred that we pay a visit and see for ourselves. The school director said to the researchers that when he was the director of a hospital many people proposed to him projects whose implementations were practically inconceivable. It seemed that the implementation of a distance education program in that school was itself inconceivable. This position is similar to those expressed in developing countries which lack the technological infrastructure and communication systems that are constantly broken-down by strife (Connell, 1998).

The data obtained from interviews with teachers and directors showed that training needs of teachers were always there and that many training bodies were in charge of training teachers to accommodate for the educational technologies proposed by the New Framework for Education in Lebanon. Connell (1998), who suggests that highly skilled staff need to be trained in the use of new technology, shares this finding. Although a number of interviewees, mostly teachers, expressed grievance against the training sessions they received, additional training was not really regarded as a priority. A teacher in a rural school reported that the training she received brought no news. A key informant, in charge of training at the Center for Educational Research and Development (CERD), said to the researchers during an interview that there is much chaos in the training and many schools in North Lebanon started to question the worth of the training session they received. Fifty percent of teachers agreed versus 28.6% disagreed with the fact that there is a shortage of qualified trainers who will carry out the cycles of teacher-training in Lebanon. Despite teachers' views, a key policy maker had the faith that teacher-training programs with a new thinking to be introduced an objective worth striving for.

Another example of the wariness of the idea of distance education was obtained from a school situated in Beirut. To a question on the school's readiness to endorse a distance education program, the school director reported that he did not have classes. When the researchers asked him about teachers' interest in pursuing higher education through distance he repeatedly mentioned that his school did not have regular classes for teachers to have the time to pursue their education. In view of the interview conducted by the researchers with the school director, that by limiting the discussion to lack of teachers and classes in the school, the director was pushing the idea to a deadlock and probably he was aware of it. Teachers, on the other hand, reported that if courses would fit within their timetable, they would take more courses leading to a degree. Even a distance education program would save them lots of time that will able them to register in a program if approved by the Ministry of Culture and Higher Education. The mean on these items ($M=5.28$, $SD=1.18$) was higher than the middle point. However, this agreement did not reach significance ($t=1.09$, df=111, $p>0.05$). To some extent, this finding corroborated O'Malley et al. study in which teachers felt that they were to benefit from instruction through Internet services especially that distance education would save them lots of time.

The positive assessment of distance education by teachers reflected a different perspective than that presented by school directors' perspective on physical abilities of their school to accommodate for distance education. Judging by what one school director said the physical conditions of his school couldn't accommodate for installing computers needed for distance education. The pertinent literature discloses that many schools operate aging computers in places as the US and Australia (Becker, 1994; Shearman, 1997). In principle she favored the idea of distance education but added that the matter concerns teachers, who in her opinion, may not have the time and money to pursue higher education through the Internet. Their main interest, she assumed, was seeking extra jobs for extra money generation.

This study has presented uncertainty regarding the implementation of distance education in schools. This uncertainty can be assorted into the followings: (i) there is wariness to the idea of distance education; (ii) the idea of distance education was seen as a relatively new one; (iii) economic and physical constraints are not conducive to the implementation of distance education program (Interviews with school directors). An overriding caution permeated those school directors who thought that distance education may keep teachers from performing their daily duties. A different picture is presented by Cuban (1986) as an important construct to a fundamentally more traditional picture that appears in Lebanon. He observes that there is a cycle of technology promotion advocated by administrators and researchers in the West because teachers are under-trained to use technology with little funding for meeting their training needs. In Lebanon this picture appears to be at variance. Administrators think of distance education as inconceivable, while teachers are neutral about its implementation or workability but feel that training is a necessity.

Notwithstanding the dry results of the study, there are no facile generalizations that are very helpful to conclude that the idea of distance education is inconceivable. Our results may not reflect the same strictures in other schools in other countries in the Middle East since these schools may have their own sociological, demographic and pedagogical characteristic features. The fact that the results are only confined to the sample of the study, nevertheless, facts presented in the study are essential to establish a framework for futuristic action.

Recommendations

The idea of promoting distance education in schools in Lebanon should take into consideration the following: First, sufficient attention has to be given to school realities. More recognition of teachers' training needs and ability to participate in distance education should be explored. Workshops recruiting school directors and teachers may present the agenda of distance education, clarify the matter and answer participants questions and queries. Second, the idea of distance education needs marketing. The distribution of a prospectus about the program, its pedagogical philosophies, and objectives can be helpful in educating schools about the serviceability of distance education. Third, presentations delivered in schools by tutors from the Departments of Education in universities about the nature of distance education can be helpful in crystallizing the idea and marketing it. Fourth, follow up meetings with schools can promote the idea and make it more discussable.

Generally speaking, our results do not rule out the possibility of implementing distance education programs in schools in Lebanon, nor it recommended its immediate implementation. The main suggestion presented in the study was that overt unfamiliarity with distance education by respondents has been a factor chiefly responsible for their lack of sustenance for the idea. The respondents' questioning of the applicability of distance education in their schools are least likely to taint a futuristic project if further steps for familiarizing them with its tenets were considered.

References:

Abdul Kargbo, J. (1997). The Internet in Sierra Leone: The way forward? First Monday, 2(2) *http://www.firstmonday.dk/issue/issue2-2/kargbo/index.html*

Becker, H. (1994). Analysis and trends of school use of new information technologies, *http://olympia.uci.edu/EDTechUse/TEXTCONCL.HTM*

Boli, J. & Ramirez, F. (1986). World Culture and the International Development of Mass Education. In J. G. Richardson, ed., *Handbook of Theory and Research in the Sociology of Education* (65-90), Westport, Conn.: Greenwood.

Bybee, R. (1993). Reforming Science Education. New York: NY: Teachers College Press.

Cohen, L & Manion, L. (1994). Research Methods in Education. London: Croom Helm.

Connnell, T. (1998). Distance holds the key growth. Times Higher Education Supplement, 1341, 12.

Czerniak, C., Lumpe, A., Haney, J., Beck, J. (1999). Teachers' beliefs about using educational technology in the science classroom. International Journal of Educational Technology, 1(2) *http://www.outreach.uiuc.edu/ijet/v1n2/czerniak/index.html*

Van Kourt, R. (2000). Providing content and facilitating social change: electronic media in rural development based on case material from Peru. First Monday, 5(2) *http://firstmonday.org/issues/issue5_2/vankoert/index.html*

Keegan, D. (1980). On defining distance education. Distance Education: An International Journal, 1(1) *http://www.usq.edu.au/dec/decjourn/v1n180/keegan.htm*

Lewis, L., Farris, E., and Alexander, D. (1997). Distance Education in Higher Education Statistical Analysis Report. *Postsecondary Education Quick Information System.* (Report No. NCES-98-062). District of Colombia (ERIC document No. ED 413 829).

Mathews, D. (1999). The origins of distance education and its use in the United States. *THE Journal Technological Horizons in Education*, 27(2), 54-68.

O'Malley, J. & McCraw, H. (1999). Students perceptions of distance learning, online learning and the traditional classroom. *On-Line Journal of Distance Learning Administration*, 2(4)

McLaughlin, M. (1990). The Rand change agent study revisited: Macro perspectives and micro realities. *Educational Researcher*, 12, 11-16.

Radford, A. (1997). The future of multimedia in education. First Monday. 2(1), *http://www.firstmonday.dk/issues/issue2_11/radford/index.html*

Rogers, E. (1995). Diffusion of innovations. New York: *The Free Press*

Schuttoffel, M. (1998). Reflections on the dilemma of distance learning. *Intenational Journal of Educational Telecommunications*,4(1) http://www.ace.org/pubs/ijet/v4n1.htm

Shearman, A. (1997). How can the effective use of computing be promoted in classrooms? Electronic Compute-Ed: *An Electronic Journal of Learning and Teaching with and about Technology*, 3 http://www.aace.org/pubs/ijet/v4n1.htm

Sherry, L. & Morse, R. (1995). An assessment of training needs in the use of distance education for instruction. *International Journal of Educational Telecommunications* (IJET) 1(1) http://www.ace.org/pubs/ijet/v1n1.htm#Sherry

Singh, G. (1993). Equality and Education. London: Albrighton.

Tam, S. (1999). Developing countries and the future of distance education and open learning in the twenty first century. *Journal of Instructional Science and Technology*. 3(1), http://www.usq.edu.au/electpub/e-jist/article3.html

Tobin, K., Tippins, D., & Gallard, A. (1994). Research on instructional strategies for teaching science. In D. L. Gabel (Ed.) *Hanbook of research on Science Teaching and Learning* (pp. 45-93). New York: Macmillan.

Yelland, N. and Bigum, J. (1995). *Professional development model for primary teachers participating in a computer technology for schools*, Australian Educational Computing Educational Council for Computers in Education, 10(2), 24-26.

Appendix

Distance Education Attitude Scale

Section I

Sociodemographic Information

1. Date of birth: Day |_|_| Month |_|_| Year |_|_|_|_|

2. Please tick the box: Male ☐ Female ☐

3. At What level are you currently teaching?

Elementary ☐

Middle School ☐

Secondary ☐

Others specify ____________________

4. Degree held:
BA ☐

BA/Teaching Diploma ☐

MA ☐

Other:____________________

5. Your Major:____________________________

6. Number of years teaching: ______

7. Are you acquainted with distance education? Yes ☐ No ☐

8. What subjects are you currently teaching? (list below)

9. You are asked to provide your own believe of the effectiveness and worth of distance education. Keegan (1980) has defined Distance education functionally: Distance Education allows the dissemination of information in which it does not require the physical presence of an instructor, in some cases were the presence is needed it requires an extensive application of organizational principles through technical media to produce high levels of instructional material. You are to rate the items for a low of (1) meaning you disagree to a high of (7), meaning agreement. Please tick in the cell (✓)

	1	2	3	4	5	6	7
1. Most people believe that distance education is more effective than traditional methodologies							
2. I think I will learn better in a distance education course than a traditional teacher-service course							
3. I would benefit from a nationally accredited distance education course.							
4. Distance education courses will work with my schedule							
5. It would probably be very difficult to contribute to class discussions through distance education.							
6. Distance education will probably allow me to take more courses than in the traditional courses.							
7. Distance education courses will save me lots of time.							
8. There is not enough money to implement a distance education program.							
9. Distance education program will not work in countries like Lebanon and the Middle East.							
10. There is a lack of trained people who will manage and run a distance education program.							
11. The school I am working in will accommodate for a distance education program.							
12. I will enroll in a distance education program if the Ministry of Education accredits it.							
13. I value a traditional university degree more than a distance education one.							
14. I do not think that a distance education program will be as rigorous as the traditional university program.							
15. I will be able to get job in Lebanon after I graduate from a distance education program.							
16. It would be very difficult to see who did what and in what way through a distance education program.							
17. I will be able to contribute in a distance education program through instruction or preparation of material.							

A Teachers Experience in Developing a Set of Interactive Computerized Tests.

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ABSTRACT

A team of teachers worked on a project. The output of which was a set of interactive Math tests delivered on a CD-ROM. The aim of the project was twofold. First, it was meant to give to the team members a chance to learn a new technology – use of ToolBook – and apply it in teaching. Second, it was meant to create an educational tool that would expose students to a different type of learning: computerized formative evaluation.The paper includes a description of the experience as a whole. The paper also includes the reflections of the author on the experience itself: When the developer and the teacher meet.

Overview

Every year, the Quality of Teaching and Learning (QTL) Program, a part of the Higher Colleges of Technology (HCT) makes a call for projects. The QTL Program seeks to enable the simultaneous achievement of enhanced quality in student learning as well as professional development experiences and outcomes for academic staff. In 1998, the priority theme was Student Assessment. Three teachers from Abu Dhabi Women's College, Nada Alameddine (Math teacher), Lina Daouk (Computing teacher), and Hanan Zaki (Computing teacher) decided to make a proposal for the project. The later was accepted. A year later, Nada Alameddine and Anne Ginneste (Business teacher) made a proposal for a continuation of the first project; it is now in its final stages.

Project Aim and Rationale

The original aim of the project was to improve Certificate Diploma (CD) students' mathematical skills. It was intended to achieve this aim by giving them a chance to exercise and self evaluate: Term 1 to Term 5 CD students could go to a Math Web site where they could choose the mathematical topic along with the level of difficulty. The interactive setting would move them from application all the way to analysis situations.

Adjustment to the Original Project

Due to time and software limitations some changes had to be implemented. The aimed target, CD students from term one to five, had to be limited to Term 1 only. In the second project, another set of tests was prepared for Term 2. The computer program to be used was also changed; ToolBook Instructor II was the most suitable one due to its advanced features. After preparing the books (this is how every test is called using ToolBook) for the first project, scoring the tests was not working properly and had to be disregarded. Publishing the books of term 1 Project on the Web was replaced by packaging and then burning them on a CD, some of the questions were not publishable. The Term 2 project however, has the scoring option. It will also be packaged and burned on a CD and published on the Web as well.

Project Phases

Description of intended clientele

The project is intended to be used by students in their first year of college. They are CD students. They have finished their secondary school successfully but are very week in English and in Math. They have studied their Math in Arabic and have mainly been exposed to rote learning.

The team members

All the team members are teachers at the CD level. Although they were all computer literate, none of them had any knowledge of the possible computer programs that could be used to execute the project. Several workshops were attended to help in the choice of the best program.

Choosing the appropriate program

This phase included exploring several programs: FrontPage '97, Question Mark Design, ToolBook Assistant II, and ToolBook Instructor II. The later had the most capabilities. A lot of the features of ToolBook were discovered in the process of constructing the books.

Mathematical content and tests writing

The Math Program was studied. Tests were written to cover all the objectives. In appendix A, you can find an organizational chart of the topics covered in Term1. The questions were written as simply as possible to meet the language level of the students. Some of the questions were at the knowledge level, some at the application and a few at the analysis level.

Objectives

For Term 1, it was decided to prepare two tests for every mathematical objective. The first level contains a set of direct questions while the second one contains questions at a higher level. Each question has a correct answer option to be checked by the students when needed.
For Term 2, it was decided to prepare one test per mathematical objective with answers and at the end, a scoring option for every test.

The project design

All the books in a project are connected to a Menu Book were students can make a choice of topic and level. Books on the same topic are connected to each other without passing by the Menu book.

Designing a master Test Book

Every test is written in a separate book. It was decided to prepare a master test book that includes different types of questions on different pages: multiple choice, fill in the blanks, matching and true false. A standard page design was worked on: background, colors, places of text, questions, linking bottoms...

Developing the Test Books

In the first project, since the team was learning ToolBook while developing the test books, a lot of effort was put to get the desired outcome. In the second project, this part went smoothly, most of the problems were anticipated and dealt with in the master Test Book.

Testing and feedback

Testing was done on several stages: first by the team members, to check on the operational mechanics of the books, then by a Math teacher, to check on any mathematical errors in the questions or the answers. It was also checked by an English Editor to check on the appropriateness of the English level.
The next step was piloting the projects. First with a couple of students and then with hundreds of students from different colleges. Small changes were applied after piloting. The final version of the Term 1 project will be used in September 2000. The Term 2 project will still have to go through the piloting in the whole system in November 2000.
A questionnaire was filled by most of the teachers who used the first project with their students. The feedback was very positive.

Benefits

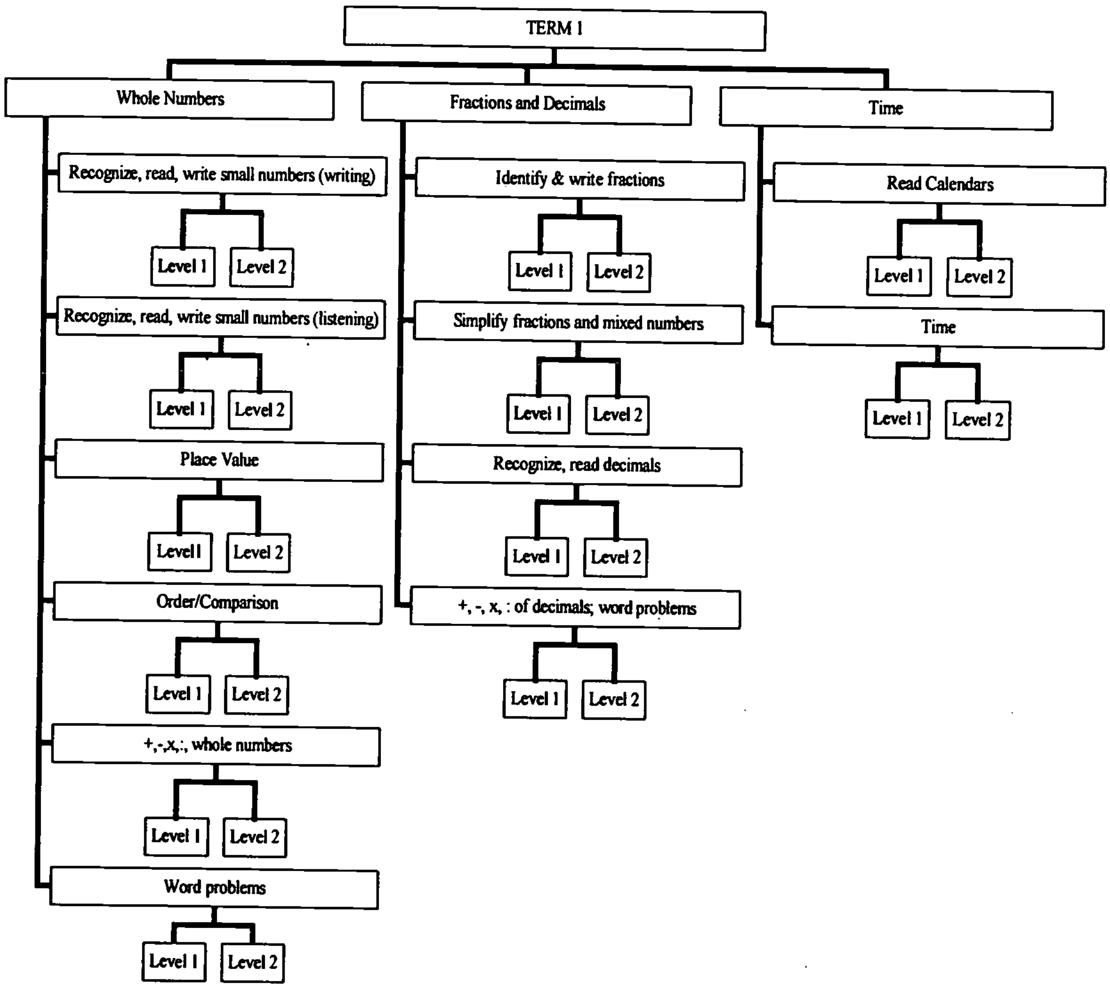
The tests developed are a unique way for the students to use away from the traditional methods practiced in classrooms. These ToolBook tests can give students not only interactivity but also responsibility, something Term1 and 2 students are not used to. Students using the books, have to make many choices like deciding on the topic as well as the level at which they want to be evaluated at. Some of the questions are at a higher level than what they are usually exposed to in class. This added some challenge to the better students. The weaker students benefited from the fact that they were working individually and taking all the time they needed to answer the questions.
For the team members the experience was very beneficial. Working as a group, exploring different computer programs, taking different decisions, designing the books and their connectivity, testing the books, debugging the errors, making changes and a lot more was learned from working on the project.
For the HCT, one of the benefits is the availability of a new learning tool that could be used as a class activity or as an individual learning activity. Another benefit is the possible sharing of the team members' experience with other teachers and helping them to work on projects in other areas.

When the Developer and the Teacher Meet

The experience was enriching but stressful. Compromises had to be made regularly between how the Math question should be and how the computer program could deliver it. An important issue was how much time and effort the team was ready or even capable at certain times of putting in order to make a test look in its best form. A small detail took hours sometimes to get. It was very frustrating to spend days before finding the source of an error, and then having to make major changes to solve the problem.
What was interesting, however, was how addictive it was becoming to work with ToolBook. Once you try to get to a point, you see more interesting features; it gets more difficult, but also more challenging. You know you will have to stop at a certain time if you want to finish within the deadline.
What makes the experience rewarding is that you are getting exactly what you want for your students. A lot of Math software is now available in the market, you can also find a lot on the internet, but in our case, the student needs were so specific that nothing is as good as the output of the projects.

As a teacher I am ready to repeat the experience, it was worthwhile.

Appendix A



Some Current Developments in the Production and Application of Interactive Mathematics Teaching/Learning Modules at the Higher Colleges of Technology in the UAE.

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ABSTRACT:

This paper reports on current developments in computer based teaching and learning in mathematics at the Abu Dhabi Men's College. The authors describe some of their experiences in the production of interactive modules using the authoring package Toolbook Instructor. The drive to develop these modules has been due mostly to the absence of learning materials, which are both culturally relevant and easily understood by students, whose first language is not English. The paper discusses the mathematical background of students entering the college and describes how the modules have attempted to address some of the students' learning difficulties in mathematics.

The Higher Colleges of Technology (HCT) is a system of post-secondary colleges for nationals of the United Arab Emirates (UAE). It was established, starting with four colleges in 1988, and this has now extended to a total of 11 colleges throughout the Emirates. It provides students with vocational and technical education in Engineering, Business, Information Technology, Health Science and Communication Technology. Students are currently able to enter into four types of programs: Certificate, Diploma, Higher Diploma, and the Bachelor in Applied Science. The Certificate programs are two years in length. They introduce students to general and specific occupational skills and develop basic proficiency in English, computing, and mathematics. The Diploma requires a further year of study in which English proficiency is further developed and occupation-specific skills at the technician level are emphasized. Students following a Higher Diploma (HD) program are required to successfully complete a one-year Foundations course before being permitted to enroll in HD. The HD programs are three years in length and involve a combination of theoretical knowledge and practical applications at the technologist level.

All classes are delivered in English, and entry to programs is dependent both on high school performance and ability in English. As entry standards to the HD programs are higher, students in the Certificate and Diploma (CD) are generally the weaker in English. All students are native Arabic speakers and are required to successfully complete all their courses in English. For many CD students, being taught in English, working in a relatively new type of educational environment, and having to become accustomed to both male and female expatriate teachers, are additional barriers to be overcome. Teachers therefore have to be constantly looking for ways in which to assist students in making the transition from students' previous traditionally-based classroom experiences, where Arabic is main mode of instruction, to the more student-centred HCT learning approach using the medium of English.

Many of the learning difficulties in mathematics that we have identified in our students at HCT, particularly at the Foundations and CD level, are closely related to limitations in the English language. These naturally lead to misinterpretations and confusion. As teachers of mathematics, therefore, we need to be pay particular attention to vocabulary and to keep language as simple as possible while still maintaining the development of concepts. This means that much of the mathematics taught to students in their first year at HCT goes back to basics of arithmetic and algebra, with the emphasis on developing English terminology and reinforcement of prerequisite concepts. The use of pictures, illustrative diagrams and concrete materials play an essential and important part in this development. The language and terminology associated with a particular mathematical topic can be quite considerable and great care has to be taken to ensure that each of the related mathematical concepts are treated with a meaningful approach to students. Sensitivity to the social and cultural background of students would bring further relevance and meaning to the students' learning.

To further enhance student success, the HCT revised its educational model in 1998 to reflect current trends in learning-centred education with it new learning paradigm. This has brought about a shift from providing instruction to producing learning, with the creation of independent learning centres (ILCs), custom designed labs for integrated learning and increasing use of technology and the internet. In a qualitative study at the Abu Dhabi Men's College (ADMC), Kahler (1998) reported that CD and Foundations students welcomed these new approaches and facilities, particularly independent learning activities. Students' comments were quite favourable and indicated that they were indeed learning more effectively and with more enjoyment.

Students should have the opportunity to investigate the mathematics through a wide variety of examples which would also help to explain and reinforce the various terms and words associated with given concepts. These examples need to have a hierarchical structure, allowing the student to move gradually through stages of difficulty levels from simple to hard. Williams (1995), who has taught both mathematics and physics in the Gulf for many years, states that the student will understand the subject best by developing a good step-by-step problem-solving technique and by learning to use simple diagrams to illustrate ideas when words can be very limiting. The teacher should encourage the class to experiment with different ideas and steps before they can make a final decision on which solution strategy is the best and most efficient. In this way he/she will learn to represent word problems symbolically and develop problem-modelling skills which are transferable over a wide range of fields. (p.1)

While there are many excellent textbooks for learning mathematics, many of these are written for Western students and not really suitable for our students in the UAE. Texts have been, and continue to be produced in-house for courses at the HCT, but these too, due to the great diversity of staff, both in experience and origin, are constantly being modified and updated. They are nevertheless beginning to find their level of usefulness as suitable student texts. Unfortunately, the majority of the texts are designed for classroom work with the teacher as a guide and facilitator, and they serve less as independent learning modules. With the availability of excellent computing facilities at the HCT, a group of mathematics teachers at ADMC embarked on the production of computer-based interactive learning modules which would provide opportunities for independent learning and meet objectives of the learning paradigm at HCT. The group started with two projects, one for CD Business

Mathematics, and one for CD Technology Mathematics. An investigative approach was used with a learning hierarchical design.

The software design objectives for the modules were follows:

- Write and design using language which is both simple and meaningful in the local UAE context to bring more relevancy to the students' learning experience to supplement the conventional teaching and learning.
- Enable students to interact with the modules at their own pace allowing them to revisit and reinforce specific concepts which will be supported with examples and practice exercises providing immediate feedback.
- Provide students with immediate feedback to their responses. This is central to understanding, enhancing learning and providing guidance and motivation. Students are able to experiment with a variety of exercises until the learning concept is clear. Include "Show Me" and "Try these" paths for in-depth learning and investigation.
- Structure the learning modules to enable the student to select "Look Closer" paths in which he/she controls the depth to which the learning proceeds. Students can therefore concentrate their efforts on those areas in which they are having particular difficulty.
- Use the audio-visual medium to provide a 'concrete-level' of thinking to supplement the slightly abstract concepts that the student will meet. This can be achieved through carefully designed animations and selected clipart. This approach will be of much benefit to mathematics students at the CD Level, particularly as they are learning in a second language.
- Further enhance motivation and bring enjoyment to the learning environment through the use of Active-X control characters (Merlin, Genie, Oscar and Big Al). The ability of these characters to talk and communicate with the student strengthens the two-way interactive learning process. Students have a desire to work with computers and may find more enjoyment in this form of interaction. With enjoyment, hopefully there will come higher motivation and learning.

Mathematics teachers from around the system have responded well received to these modules with several planning to implement them into their teaching. Some have made suggestions for improvement as these are expected to be incorporated in the package. Through the design and development of the modules, the team has gained some insight into teaching approaches to UAE students learning mathematics in English a second language. It was important to ensure that the modules did not simply correspond to simply reading and turning pages, effectively replacing a textbook. The multimedia opportunities afforded by the authoring software were carefully explored and implemented to provide good clear instructive learning. Toolbook Instructor was found to be an excellent medium for both design and giving students a large measure of control over their learning process. Animation, sound, and a variety of techniques were used to gain student attention and provide a positive impact on learning and problem solving (See Appendix). Students at the HCT have no computer anxiety and show favorable attitudes to working with technology based learning environments. They make regular use of the internet as a source of information for project work. On-line courses are currently being developed.

The mathematics modules described here are now installed in the ILCs around the HCT system of colleges. Some teachers have started using them with their students. While no research has yet been undertaken to examine their effectiveness, there has been much positive and favorable feedback to date. Until further investigation on their effects on learning can take place, the authors expect that they will provide

- weaker students with the opportunity to learn at a pace more suited to their level of understanding
- motivation to students in thinking more about the mathematics
- help to students in developing appropriate learning techniques and investigative thinking
- a supplement to conventional classroom presentations
- remedial work for students as seen necessary by the teacher
- students at higher levels of their programs of study with a review source of fundamental concepts

We need to remember that only about twenty years ago, there was no formal education for 85% of the UAE National population. Today the UAE is producing graduates with a sufficient understanding of concepts, principles and skills, and who are contributing to the general development of the country. Students at the HCT now have access to learning centers equipped with current computer technology and it is up to individual teachers to adapt their teaching approaches to make good use what is available. In addition to developing the software described in this paper, existing computer packages such as *Derive* and *Plato* are being used to assist with student learning in mathematics.

References:

Kahler, W. (1998). ADM students' perceptions of the quality of their new learning experience. *The Higher Colleges of Technology Journal.*3 (1), 3-15.

Williams, B. (1995). *Effective teaching strategies - math and physics. HCTech,* March, 1-2, HCT In-house publication.

Using Technology as a Tool for Teaching Mathematics at the Secondary School.

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ABSTRACT

The purpose of this paper is to share our experience at Saint Mary's Orthodox College concerning the use of technology as a tool for teaching mathematics in the classroom. Even though we are implementing the new Lebanese curriculum and are limited by several constraints still, we were able to integrate technology in our classrooms and in many cases it helped us integrate math concepts together. The paper will include our teaching plans in a Technology Based Environment, as well as some preplanned activities with students' feedback.

Curriculum and Technology

The purpose of this paper is to share our experience on the use of technology in teaching mathematics at the Secondary school while applying the new Lebanese curriculum. It reports our work in integrating technology as a tool for developing imagination and creativity. On the other hand, technology serves to maximize the benefits and offers students the time to develop many skills.

The new Lebanese curriculum emphasizes connecting mathematics to real life, on one hand, and integrating technology in the teaching-learning process on the other. But the content that needs to be covered is too big. In our first encounter with the new curriculum, we used the technology of specialized software and graphic calculators in order to achieve the general objectives of the curriculum and save time.

Our experience in using specialized software is not a new one, and this springs out from our belief of the chances that a technology based environment (TBE) provides to students. It is no doubt that TBE offers to students the chance to contribute in building concepts and possibilities to explore different cases which are impossible to be realized with paper and pencil. "To see something once is better than hearing about it hundred times", that was the essence of our experience. Through the "eye" of the graphic software, students were able to visualize mathematical processes and concepts and explore them from multiple perspectives: graphic, numeric and symbolic.

The main challenge that TBE has resolved is making abstract notions (like functions) concrete and tangible.

TBE offers students the opportunity to explore, conjecture, verify, try many cases, measure, classify, doubt, discover mathematical methods and later formalize. Furthermore, in an activity done in a TBE, learners would no longer be limited to the content of the activity but have the chance to explore other meanings and connections of the content. This opens new horizons for the learners, which consequently widens the spectrum of their mathematical thinking. TBE, as well, proposes a new way of communicating mathematics, a way full of visual and intuitive aspects. It puts the students in the position of a mathematician who builds his thinking inductively through observation, exploration, development of understanding before making his mathematics formal.

Methodology

The TBE that our experience has created in teaching algebra and trigonometry has offered the students opportunities for experimental activities and inductive reasoning which ultimately aims at stimulation of interest in mathematics as a whole. In this paper, we will present 3 introductory activities done with DERIVE. The purpose of these activities is to offer student the environment in which they are to explore new concepts, make links with previous knowledge, and integrate Calculus in the study of Algebra and Trigonometry.

Our work for these sessions was done in a special classroom equipped with computers, a white board and an overhead projector. Students were divided into groups, computers were used upon need and according to the directions of the activity.

The following activities were given in grade 11 Sciences section in different stages of the academic year.

- ***Activity 1: Deducing the graph of sin(x) from the graph of cos(x)***

A real understanding of graphs makes clear several important things about the variation of these graphs. In particular, there is a general similarity in the shape of the graph of any trigonometric function and its corresponding co-function. In fact, one can be obtained from the other by translation.

The activity consists on making students explore the variation of the cosine function and then on asking them to deduce the graph of the sine function.

This activity was carried in to different classes, in two different ways:

In the first case, students observed the variation of the cosine function with Derive. Then, they were asked to deduce the graph of sin (x) from that of cos (x). When they could not, they were notified by their teacher to make use of trigonometric relations. Some were able to use cos ($\pi/2 - x$) = sin (x) and to deduce that one of the graphs is a translation for the other by $\pi/2$.

In the second case, we changed our strategy: students observed the variation of the cosine function with Derive. Then, they were asked to deduce the graph of sin (x) from that of cos (x). When they could not, they were asked to plot the graph of sin (x) and to answer the same question. It was so easy for them to talk about translation that was justified by the use of trigonometric relation: cos ($\pi/2 - x$) = sin (x).

- ***Activity 2: Differentiating between sin(kx) and ksin(x)***

Functions such as sin(2x) and 2sin(x) frequently create great confusion. And it is important some special attention to the interpretation of functions expressed in forms such as those indicated above.

Comparison of graphs, in this case, points out the fact that in the first case the constant is a multiplier of the argument and so affects the period of the function but not the amplitude; while in the second case the constant is a multiplier of the function itself and so affects its amplitude but not its period. Students realized the above effect after observing graphs with different values of multipliers after checking many values. This type of work was easy in an environment like DERIVE, where many cases can be checked within a limited period of time.

In the following figures, we can see some of the variations generated by DERIVE for this activity.

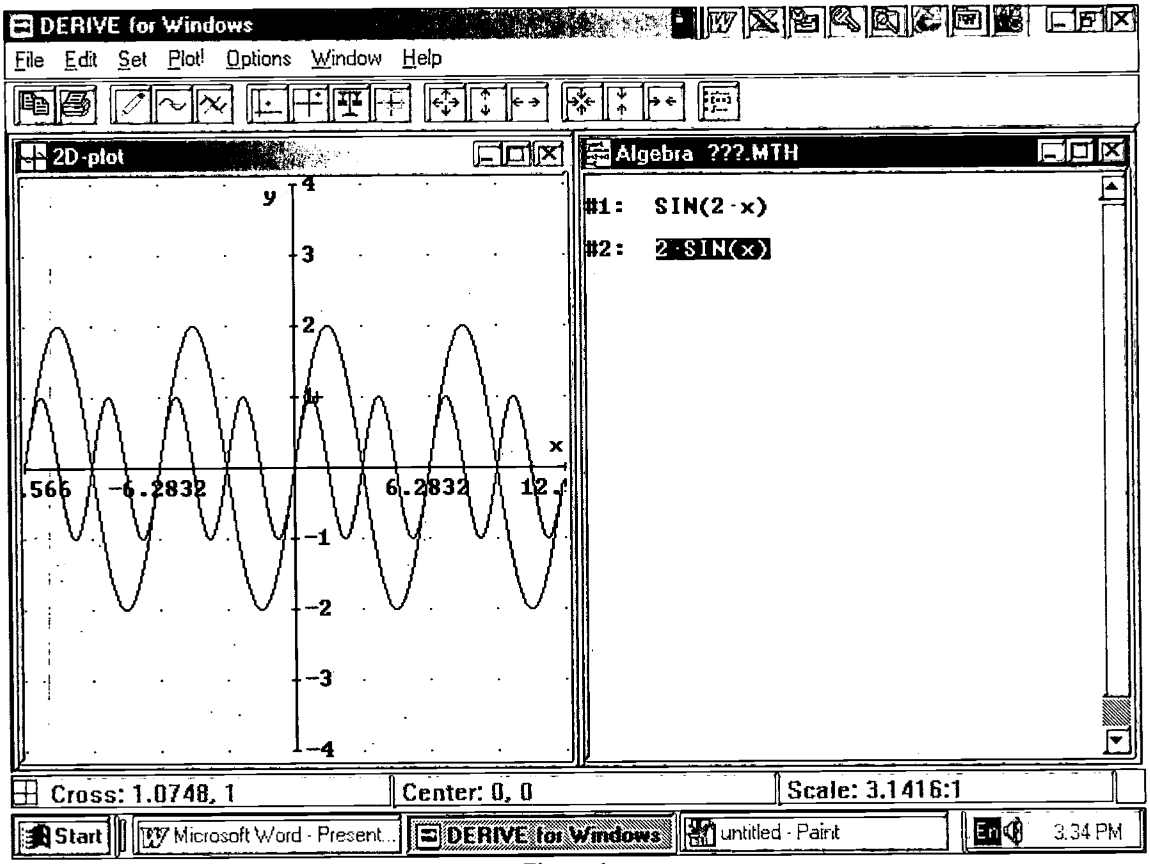

Figure 1

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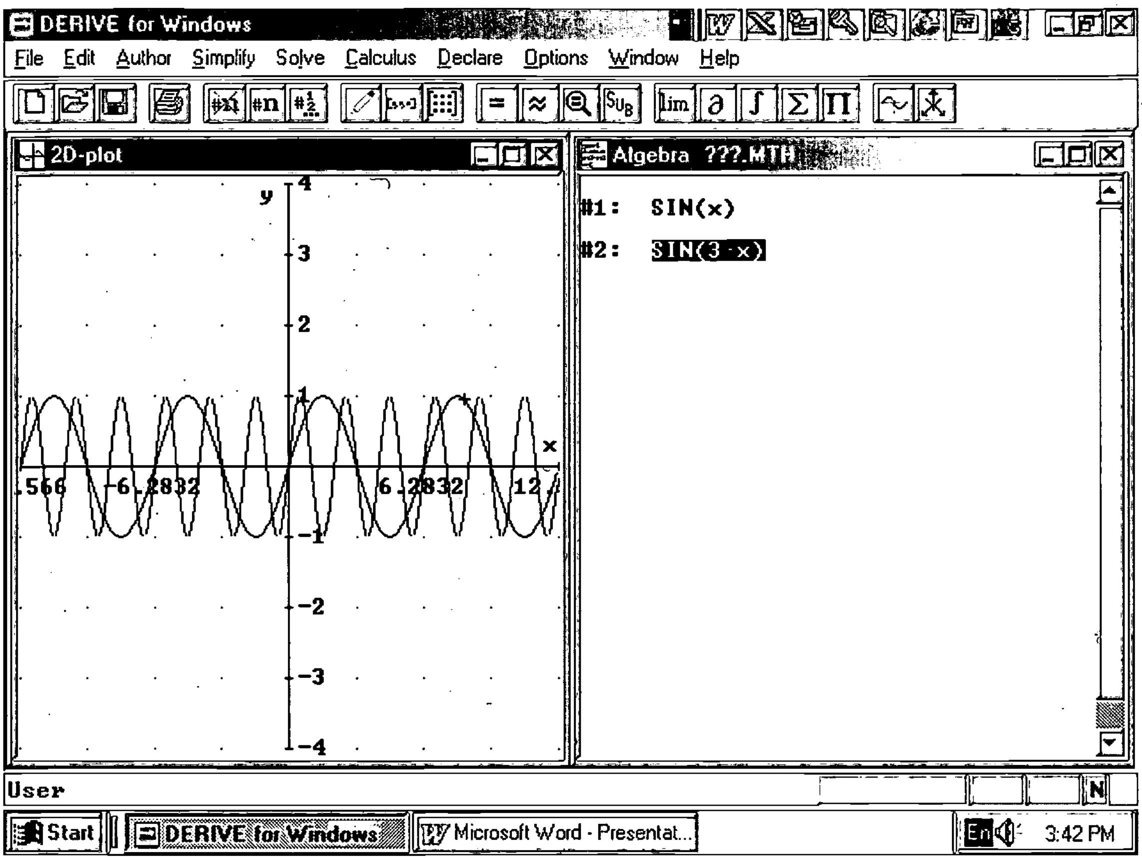

Figure 2

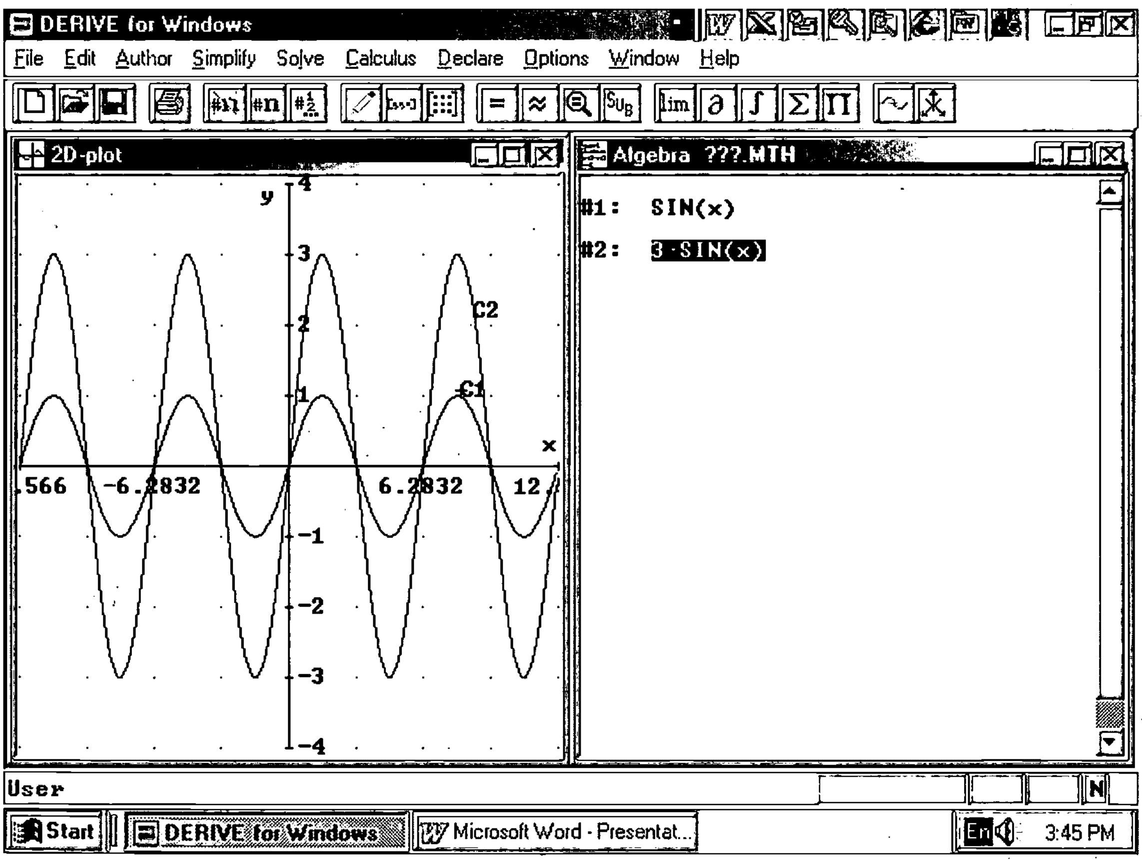

Figure 3

- ***Activity 3: Study the variation of functions***

Students were exposed to different types of graphs on which they had studied the concept of limits, discover by that the meaning of an asymptote (graphically) and accordingly were able to grasp the algebraic sense of a function. Students were exposed to different types of graphs on which they had studied limits. The concept of asymptote was then easily introduced and students were able to define asymptote themselves, this process went smoothly. All of this helped in studying the variation of a function and in plotting graphs of rational and irrational functions later easily.

Conclusion:

This is our experience in using technology in teaching mathematics. Today we are more sure that TBE:

- Emphasizes the unity and interconnectedness among mathematical ideas.
- Touches on different skill levels and interests.
- Enhances reasoning and problem solving in new and varied ways.
- Promotes conceptual development, challenge skills, excite curiosities and stimulate imagination.

We are trying to optimize our students' thinking abilities. We, as educators, are interested in teaching our students' problem solving rather than solving problems. It is a big problem if our students know how to write a proof but unable of generating one.

The Multimedia in our Mathematics Classroom

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ABSTRACT
Imagination and movement mark adolescence...(Piaget).So why should one be against nature ? Multimedia, throughout the new technology of information is an undeniable and a powerful tool in such a domain. We will develop in this paper:
1) the influence of the new technology on each component of our educational system (student - professor- knowledge)
2)How we converged to a new concept of Methodology based on a dynamic approach of concepts

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Historic outline

If the printer marked the school in its educational practice, see Freinet [1], at the beginning of the twentieth century, the new technology of information and communication : computers, internet, multimedia ...is going to change the educational system radically at the end of this century.

Democratization of these products

The democratisation of these new technical supports allowed a transverse profit. Its use doesn't remain anymore a privilege of industrialized countries. Now, this technology of media extends firmly in developing countries. In a country as Lebanon it's rare to find a family without a subscription to internet or to find less than one computer by class in its school and universities.

A new Factor of academic failure in developing countries

Without worrying about social factors : unemployment, family problems...which are at the base of this crisis, another significant intrinsic factor in the educational system appears under the form of a conflict... Next to the preoccupations of the renovation of the system, the profit that brings us the technology is certain. Mobilization in this use has created a new school in conflict with the former one. This last one refuses the use of the calculator, even a vulgar one. It claims to have, only itself, the honour to master and to spread knowledge in a cyberworld without limit and border.

The professors are well divided between these two currents. This paradoxical situation has also a negative effect on the students and their academic failures.

Fortunately, the former current loses ground with an exponential speed in front of on one side, the flux of young people who with their innovative spirit need to adapt themselves to the new world and on the other side, the profit brought by this renovation.

Now, we are going to discuss, in brief, common modifications brought by this invasion in all the disciplines ; we will speak then, in detail, about its influence on mathematics.

Common characters in all disciplines of the influence of the new technology of information and communication

Students

On the cognitive plan.

This paradise of multimedia : colors, sound, text and their digitalization create to the young people an infinite continuation of very close situations as the inferences on the cervical level. Is that going to help the students to develop a certain intelligence close to the artificial one ?

On the sensory plan.

The lesson which is transformed into an interactive « game » between the keyboard and what the student receives on the screen is it not going to develop the sensory capacity ?

On the psychological plan.

In front of the computer, this machine which possesses a formal logic without feeling, isn't the student going to accept the criticisms of his professor or his friends at the time of the validation or invalidation of his work ?

On the professional plan.

The big screen and the limited number of devices oblige the students to work in groups. Such a realisation of tasks in groups, doesn't prepare the students to the future professional life ?

On the social plan

Are not, the exactitude and the order in which the addresses are written creating in young people's spirit, especially in not computerized societies, the need and not the will to respect some necessary rules of daily life and so, are they not going to develop a certain citizenship ?

Professors

On the professional plan

Are not, the preparation of a lesson with a computer, the visit of these different sites, so the relation between the different colleagues, the profit brought by the experience and the reflection of others, improving the quality of the professors ? Is not this sharing of ideas and concepts improving his productivity ?

On the relational plan

Is not, the professor leaving his magistrate's cap ? Going away towards his students to discover with them an asymptotic study, is not he sharing with them their successes or their failures ? Specifying the directives of search and by controlling their reactions, is not every lesson becoming a masterpiece of whom he is the maestro ?

Knowledge.

Nobody can claim to possess a total and absolute knowledge, it is always a collective production. Are not, this stunning speed of exchange of point of view and a system like « Lunix system » going to become widespread in other domain of human knowledge ?

Peculiarity of the relationship between this new technology and the « mathematics » discipline.

This peculiarity is shown in the three layers of a mathematical activity : domains, tools and intelligence.

Domains

Traditionally, the field of mathematics action included geometry and numbers. Actually, this field widens in the study of others objects, but, the *representations* of these last ones often remain at the level of Euclid's geometry in dimensions 1,2 or 3. The field of action is similar to the *directed objects* of the different programming languages.

Tools

The computer treats information according to their nature by means of three units : unit of text processing, logical unit and arithmetical one. The two last ones are based on the notion of Boole's algebra ? Are not, the three algebras : propositions provided with the two laws and/or , some parts of the set of natural numbers provided with the 2 laws $a \wedge b$, $a \vee b$ and the set of the parts of a set E, P(E), with $\cup$ and $\cap$ laws, joining this general structure of Boole ? For more details, see [2].

In this way, Michel Queyssane, in his book « Algebra » [3] said «... history of thinking shows that the elaboration of logic and the constitution of the first elements of mathematics were made at the same time and that at every moment of history, XIIth, end of XIXth and beginning of XXth century, when precision showed itself, there was a mutual action between logic and mathematics.

Intelligence

In the middle of this century the basic principle of a computer was specified by J.Von Neumann as a unity of « sequential treatment of information » ; in the fifth generation, an « operating system » quickly entered in its architecture and some years later a new discipline « artificial intelligence and expert system » was born. Without entering into a detailed comparative study between the functioning of the two intelligences, human and artificial, I limit myself to this very significant passage written by Adrien Lescort in his book [4] « Artificial intelligence and expert systems » : «... from 1957 till 1969,three American researchers, Newell, Show and Simon finalize the GPS (general program solver), a program which is able to solve problems as varied as puzzles, logical and arithmetical problems.

Its importance is to have been the first « to formalize human reasoning ». Its purpose was a search on the intellectual activity and on the mechanism involved during the resolution of problems, rather than the efficiency. The GPS marked the beginning of a current of thought in artificial intelligence which consists of saying : before making an intelligent machine, let us try to understand how we solve problems.

An expert system is constituted , generally, by three parts : the base of knowledge, the inference engine and the interfaces ».

The inference engine is a program charged to exploit knowledge to solve a problem.

Towards a new methodology of education of Mathematics

This interaction between this new technology and mathematics, at the level of the three layers of a mathematical activity (domains, tools and intelligence) shows us the need to throw back the former work method schematized by :

$\longrightarrow \qquad \longrightarrow$

and invite us to adopt the other schematisation :

Student $\longrightarrow$ Professor + Technology of Information $\longrightarrow$ Knowledge

By opening up to this new technology, and with a real will to use it in our educational practise, we face real difficulties in two different areas, tools management and epistemological tolerance.

Tools management.

The edification of the majority of schools was realized before this wave of renovation, so, the size of classrooms are incompatible with the installation of a computer by student. Let us add to this difficulty, the demographic problem, especially in big cities, the cost of these machine and their installation for a school with a moderate budget. Besides a multimedia hall, a notebook and a removable TV big screen in traditional classrooms, the use of the calculators becomes optimal !

Epistemological tolerance

How can one marry the fact that the harmonic series is convergent for a computer to the Euclid's elements ? Which tolerance can one accept about the problem of truncation imposed by the capacity of these machines, the resolution of their screen with the rigour that we need in mathematics ?

Method

The method used later considers the requirements of the second track without having the heavy consequences of the first one. We are going to baptize it « **Dynamic Approach of concepts** » (D.A.C).

- First, a presentation is realized on a big screen. Dynamic situations are developed with « generators of inferences » needed for a given concept (the first important moment of the activity). Generally, these inferences become classes of positions of one or more objects, so they help the students to discover by themselves the « inferences engine » of the problem. At this level the students frequently use their calculators, in a programmatic mode, by using modules already created or, by creating them if needed. Then, a collective writing of what was observed is done. Finally, during practical exercises periods, they reproduce, in the multimedia hall, the same presentation with a suitable software.
- The second important moment comes when the student builds in his flowchart of the inference engine and compiles it in a suitable programming language (basic or derivated, which is the closest language to that of the calculator). Let us be clear now and specify this method with examples taken from our presentation at the conference [5] and define the new introduced terms : generator of inferences, situations, classes, modules …For that, three examples are going to be chosen : the approach of the Logarithm function and its fundamental algebraic property, a geometrical construction and the search of a level curve.

Example1 : In the following two figures, we represent two situations of the variables M(x,lnx) and N(2x,ln2x). Students must realize, when moving the variable m on the real axis ox, that the distance **hN,** which is equal to ln2x – lnx, remains constant ; this one is nothing than bb' =ln2 (level : first year university, or last year high school).

fig-1-

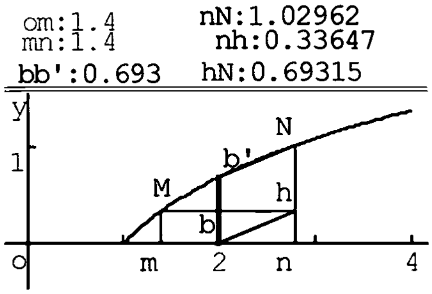
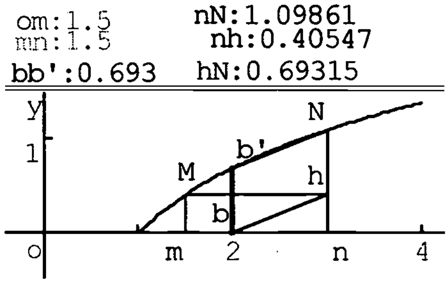


fig-2-



In this way of thinking, students should observe that the segment [hN] is an object of a class of similar ones. Here, the number « 2 » is the « Generator » of such « Inferences ». In a same way, we observe that 3,4,...are also a « generator of inferences ».

Such an observation pushes us to think that :
ln(ax) – lnx = lna for any constant a>0.

At this level, student must think that ln(ax) and ln(x) are antideratives of the same function. Experimentally we observe that the slope of the tangent ln'x is near 1/x. This Dynamic Approach helps us – professor- to introduce this function without any «parachute». The continuity of 1/x allows us to define lnx as an antiderative of it, which vanishes at 1.

Note here, that we construct the engine of inferences, by growing from lnx to ln(ax), then to ln(yx) where, now, x and y are both variables.
Example 2 : Our problem, here, is to construct a square **mhpn** such that its vertices belong to the sides of a given triangle **ABC** (level : one year before the end of high school .The students have a good knowledge on usual transformations in the plane).
Analysing the properties of such an object (square), students must discover that two vertices must belong to the same side, for example **h,p** belong to [BC], **m** must be on [AB] and n on [AC]. So the engine of inferences is formed by four generators : choose **m** on[AB], select all such objects having [hp] parallel to [BC], when the last one is on [BC] select finally from the last class the element which has **n** on [AC].We have used here four generators of inferences. It seems that such an engine is associated with a constraint. See the following four figures :

fig-3-

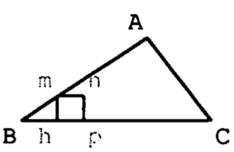


fig-4-

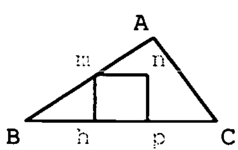


fig-5-

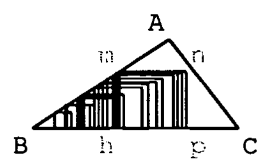
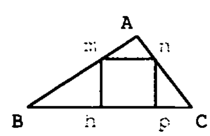


fig-6-



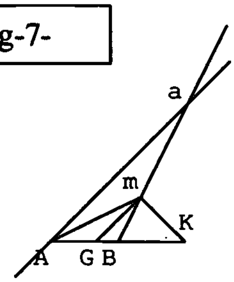
The number of generators that we have used here to solve this problem explains its difficulty for the students at this level.

Example3 : We compare here our approach to the classical one. In this example we determine the level curve **L** of the point m such that : mA=2mB where A and B are two given points in the plane (the needed level is the same as the one in example2). Let us see, now, how many parachutes we will use to arrive to the solution.

Draw the interior bisector (mG) of the angle AmB
- Extend [Bm) to a point a such that **mA=ma.**
- Prove that (mG)//(Aa).
- Deduce that GA=2GB.

For the exterior bisector we repeate the same four steps to prove that KA=2KB.

fig-7-



Let us try, now, to resolve this problem using our dynamic approach. Let us find a generator of inferences of the object **m.** The distance from **m** to A is twice its distance to B, then we can generate, in a dynamic way, **m** as the intersection of two circles centered respectively at A and B with radius 2x and x where x is a real variable verifying the condition : (1 /3)AB<x<AB. See the following three figures :

fig-8-

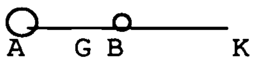


fig-9-

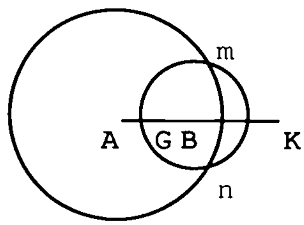
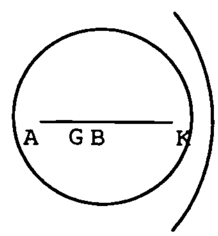


fig-10-



Students, observing **L,** recognize that it is symmetric with respect to (AB) and cuts it on two points G and K having, with A and B, the vectoriel relations :

vec(GA)+2vec(GB) = 0 (1)

vec(KA) -2vec(KB) = 0 (2)

Looking fig-10- we are oriented to prove that L is a circle of diameter [GK]then the flowchart of our engine of inferences is :

- If (L is a circle) then (its diameter is [GK])
- (L is a circle) if and only if (the scalar product vec(mG).vec(mK) = 0).

Reader, using symmetry, can confirm the first statement and by using, in a direct way, barycentric calculus from (1) and(2), he can establish the last statement. Note, here, that such calculus are in the program of this level.

Synthesis

Students' difficulties often arise in the general study of a concept, the construction of objects according to some norms, or the determination of a level line. Sometimes, these difficulties push the students to a total refusal of mathematics. The examples given above, show clearly the interest that bring us the combination of these technical supports with the D.A.C adopted. This contribution often shows itself at the level of the observation phase of an activity. The notion of generator of inferences, similar to a filter, which has been introduced will allow us, in certain cases, to go past this phase for going into the heart of the inferences engine. Are we going to converge towards a new methodology of the education of mathematics ?

Conclusion

The need of a new methodology using new information technology is not a naïve concern aiming to introduce multimedia into our educational system, only to please our students and to motivate them by animations (until we forget the interest of our course), nor a simple concern of renovation, what these machines can bring us in our discipline is a real need.

References

Algèbre, Michel Queysanne, Collection U, A. Colin, 1964.

Intelligence artificielle et systèmes experts, Adrien LESCORT,Cedic/Nathan,1985.

Presentation, Mohamed Mounir FAKHRI, International conference on technology in mathematics education, Lebanese American University of Beyrouth, 5/7/2000.

Freinet et la technologie de la communication by Monique MOINEAU, direction de l'Ingénierie Educative du CNDP, Medialog n°31, avril 1998, France.

Méthodes mathématiques pour l'informatique, Jacques VELU.Dunod 1987.

Non Trivial Applications of MAPLE in Teaching Mathematics

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ABSTRACT:

In this paper, I am trying to show how useful a very basic knowledge of Maple programming can be. This is the level of programming that can be very easily achieved, even with high-school students. I am going to demonstrate how writing Maple programs will help us to explore recursive functions and trace algorithms. I will also show how to develop some constructions for discrete mathematics and geometry. Most of the presented examples are taken from my recent tutorials for high-school teachers of mathematics and the work of my students.

Introduction:

Various mathematical programs offer an almost unlimited number of tools and options. We used to think that such programs should be as user-friendly as possible. At the same time, our knowledge of a computer program can be quite shallow. Usually we know how to solve an equation, transform a formula, and plot a graph by clicking a button. Sometimes, we also know a few basic commands, which allow us to obtain the same effects in a command line mode – if such option is available in our program. This is often all what we know about a computer package for teaching mathematics. On the basis of this knowledge, we used to think that programs where we have to use a programming language are too difficult to be used in teaching mathematics. This particularly concerns Maple V and Mathematica.

When looking at our teaching, we may notice that teaching mathematics requires many activities that are not available in click-only-to-get programs. For instance, while teaching mathematics, we have to explore algorithms, use recursive functions, or build complex constructions in 2D or 3D geometry. All these activities are built out of many steps and their nature resembles writing a kind of a program. These kinds of activities can be practiced if we use a computer program where a programming language is available. One such program is Waterloo Maple V, where the programming language is very natural and quite easy to learn.

Introduction to Maple V Programming Language:

This short paper is not intended to be a Maple tutorial. I wish to mention here only some features of Maple that can be of interest to educators. If you are going to use Maple for teaching mathematics in high school or university, the book by Cheung [1] can serve as a great and very compact tutorial of Maple features.

Maple offers a number of commands and control structures on various levels. For teaching high school students or university introductory classes you need only use a basic set of commands and control structures in the most simplified form. For teaching advanced topics or more sophisticated problems you can go into a more expanded set of Maple operations. This way, you will need to introduce your students to only a very limited set of Maple commands and control structures. In my opinion using about 30 commands and basic control structures in the most simplified form you can successfully introduce most of the high school topics. For undergraduate university courses you may need a bit more. You also do not need to introduce all these elements in one lesson or lecture. You can introduce them gradually as you need them. For example, for solving equations you can use the **solve** command as in the following examples:

> solve(x^2-2*x-5=0,x);

$$1+\sqrt{6},\ 1-\sqrt{6}$$

> solve(x^2+1=x^4,x);

$$-\frac{1}{2}\sqrt{2-2\sqrt{5}},\ \frac{1}{2}\sqrt{2-2\sqrt{5}},\ -\frac{1}{2}\sqrt{2+2\sqrt{5}},\ \frac{1}{2}\sqrt{2+2\sqrt{5}}$$

If you wish to obtain results in a decimal form you may use the **fsolve** command or just convert the obtained result to decimal form using **evalf**:

> evalf(%);

$-.7861513780I,\ .7861513780I,\ -1.272019650,\ 1.272019650$

Other useful Maple commands for introductory classes are: **expand, simplify, factor, combine, diff** (for differentiate), **int** (for integrate), **plot, plot3d, subs** (for substitute). At this level you can show your students how to assign a mathematical expression to a variable, so they can use it later without retyping a formula. This will work like this:

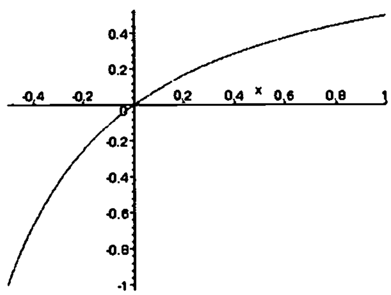
> MyEquation:=(x^2-2*x-5=0);

$$MyEquation := x^2-2\,x-5=0$$

> MyFunction:=x/(1+x);

$$MyFunction := \frac{x}{1+x}$$

>plot(MyFunction,x=-0.5..1);



Up to this moment I was following the most traditional approach to using a computer program in teaching mathematics. However you, as well as your students, may already feel that something is missing. For example the commands **solve** and **fsolve** work like a black box. You drop into them any equation and in most the cases you get a satisfactory solution. But if something goes wrong you do not know why, and sometimes even you may not notice that the obtained solution is wrong. For example why does the command

> fsolve(x^2+1=x^4);

$-1.272019650,\ 1.272019650$

produce only two solutions? Where are the two other solutions?

One of the most fundamental problems in high school mathematics is solving of quadratic equations. By introducing one or two programming instructions you can give your students a chance to explore methods of solving quadratic equation without using the black box - the **solve** command. Here you have a very simple example of Maple code that was done by a high school student and below it is the Maple output produced by this code:

```
> A:=1:
> B:=-1:
> C:=-5:
> det:=B^2-4*A*C;
 if det>=0 then
  x1:=(-B+sqrt(det))/(2*A):
  x2:=(-B-sqrt(det))/(2*A)
 else
  print("No real solutions")
 end;
```

$det := 21$

$x1 := \frac{1}{2} + \frac{1}{2}\sqrt{21}$

$x2 := \frac{1}{2} - \frac{1}{2}\sqrt{21}$

The above example is very simple and the instruction "**if ... then ... else**" was used here in a form similar to the Pascal syntax. The control structures in Maple have a very complex form, but you can always choose the form that is the most suitable for your students. For instance the general form of the loop instruction is:

|for <name>||from <expr>||by <expr>||to <expr>||while <expr>|do <statement sequence> end do;

However, you can use it in the form "**for ... from ... to ... do ... end do;**" or "**while ... do ...end do;**" or even in a very short form "**do...end do;**". Finally, instead of using "**end do;**" and "**end if;**" etc. you can use just "**end;**" like in the programming language Pascal. This feature makes Maple syntax easy to adjust to anybody's habits and needs. All the syntax examples mentioned here are for Maple VR6. Earlier versions of Maple used slightly different syntax, but also with great flexibility.
At this stage of Maple knowledge you can introduce Maple procedures to your students and you get an unlimited source of examples of functions. You are not limited to functions of one or two variables. For example, with a Maple procedure you can define a function that for any given quadratic equation will produce a set of its solutions or an empty set. This can be done as follows:

```
> CRoot:=proc(A,B,C)
 local det;
 global x1, x2, sol;
 det:=B^2-4*A*C;
  if det>=0 then
   x1:=(-B-sqrt(det))/(2*A):
   x2:=(-B+sqrt(det))/(2*A):
   sol:={x1,x2}
  else
```

```
   x1:=NULL:
   x2:=NULL:
   sol:={}:
   print("No real solutions")
  end;
 end:
```

Now you can use **CRoot** in a similar manner to the operation **solve**.

```
> CRoot(1,-3,2);
```

$\{1,2\}$

In further calculations you can refer to the obtained set of roots or to each root separately by calling appropriate variables:

```
> sol;
```

$\{1,2\}$

```
> x1; x2;
```

1

2

The above examples show that the concept of programming in Maple can be easily introduced to university students as well as to some high school classes. Some university courses will especially benefit from a knowledge of Maple programming. You can find it useful while teaching discrete mathematics, finite mathematics, number theory or numerical methods.

Exploring Algorithms with Maple:

Algorithms are major part of mathematics. Most of standard mathematical processes are algorithmic. Finding the greatest common divisor of two integers, finding a root of the equation with the use of a numerical method, calculating the determinant of a matrix, solving a system of linear equations, etc. are just algorithms used to produce a mathematical result. While teaching mathematics we concentrate on teaching these algorithms rather than obtaining concrete solutions. Thousands of examples in thousands of problems books are written just in order to practice specific algorithms. Thus using the Maple programming language students can practice how the given algorithm works and learn its tiny secrets – secrets that are often missed in the traditional approach.

While teaching discrete mathematics I often use the Euclid algorithm to calculate the greatest common divisor of two integers. This algorithm is quite simple. However for some students it can be quite difficult unless I will give them an opportunity to explore its nature by writing it in the form of a Maple procedure. Here are three examples showing how students implemented this algorithm.

```
> Euclid1:=proc(a,b)
 local u,v;
 u:=a; v:=b;
 while u<>v do
  if u>v then u:=u-v else v:=v-u end
 end;
 return u;
```

```
end:
> Euclid1(71755875,61735500);
3375

> Euclid2:=proc(a,b)
 local m, n, temp;
 m:=a; n:=b;
 while n>0 do
  temp:=(m mod n);
  m:=n;
  n:=temp
 end;
 return m
 end:
> Euclid2(71755875,61735500);
3375

> Euclid3:=proc(a,b)
  if a=b then return a else
   if a>b then return Euclid3(a-b,b) else
     return Euclid3(a,b-a)
   end
  end
 end:
> Euclid3(71755875,61735500);
3375
```

A number of other algorithms can be implemented using the Maple programming language. For instance, iterative or recursive algorithms for sorting, searching lists and other algorithms used in discrete mathematics can be easily implemented this way.

Exploring Recursion with Maple:

Recursive operations are difficult for beginners. Even experienced mathematicians often prefer to use more complicated iterative algorithms instead of applying simple recursive algorithms. Let's consider an example of a recursive function and see how it can be implemented in Maple.

> **Problem:** The sequence $\{x(n)\}$ is given by equations: $x(1)=1, x(2)=2$ and $x(n)=3x(n-1)-2x(n-2)$. Construct an algorithm to find $x(99)$.
>
> **Solution:** Using Maple "**if..then..else**" you can define a very simple procedure **MySeq** that for a given integer n will produce the n'th term of the sequence.

```
> MySeq:=proc(n)
  option remember;
  if n<3 then
   n
  else
   3*MySeq(n-1)-2*MySeq(n-2)
```

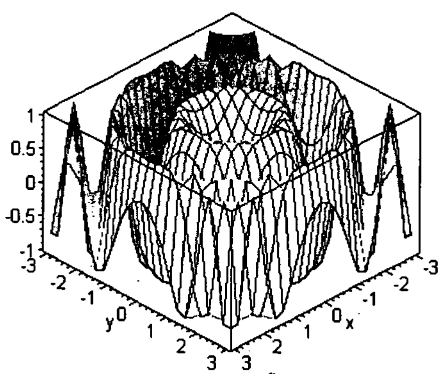
```
 end
end proc:
```
Now you can test how the procedure works:
```
> seq(MySeq(i),i=1..15);
 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192, 16384
```

If you wish to obtain the terms of the sequence in a more convenient form, you can introduce arrays and ask Maple to develop an array containing the terms of the sequence. It can be done like this:
```
> MN:=array(1..1000):
 MN[1]:=1:
 MN[2]:=2:
 for n from 3 to 1000 do
  MN[n]:=3*MN[n-1]-2*MN[n-2]
 end:
```
Now, when Maple has produced the array with 1000 elements of the sequence, you can use them in further calculations.
```
> MN[99];
 316912650057057350374175801344
```

This way you can model and explore various recursive operations in many areas of mathematics.

Visualization of 2D and 3D objects with Maple:

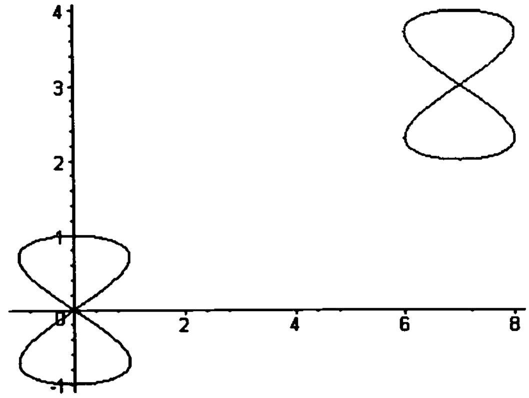
Maple can plot graphs of various objects including graphs of functions, implicit equations, lines, points and other objects of 2D and 3D geometry. Such a graph can be plotted on the computer screen or assigned to a variable for further use. For instance, let us consider a surface given by the equation $z = \sin(x^2 + y^2)$. You can assign its plot to a variable and display it on the screen at any time. Here is how it works:
```
> with(plots):
> MyPlot:=plot3d(sin(x^2+y^2),x=-3..3,y=-3..3):
 display(MyPlot, axes=boxed);
```



Having declared another plot or a number of plots of various types you can display them together using command:
```
> display({Plot1,Plot2,Plot3},[plot formatting options]);
```
The opportunity of displaying various objects together in one picture can be used to model a number of interesting investigations in various areas of mathematics (see [2]). For instance, suppose that you have a 2D curve given by parametric equations x=sin(2t) and y=cos(t) and transformation X=x+7, Y=y+3. you can easily show how

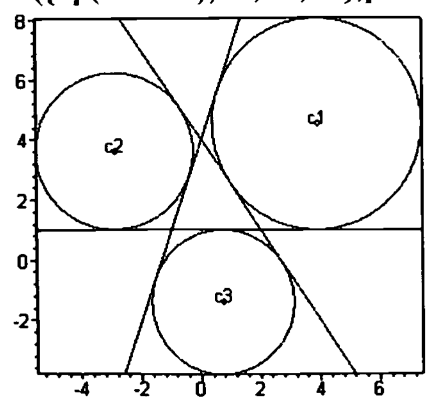
the curve looks and where is located after performing this transformation. Here is the Maple code.

```
> x:=sin(2*t): y:=cos(t):
plot1:=plot([x, y, t=-Pi..Pi], color=blue):
X:=x+7:
Y:=y+3:
plot2:=plot([X,Y,t=-Pi..Pi], color=red):
display({plot1,plot2});
```



Maple has a separate library, called "geometry", which can be used for modeling geometric constructions. Here is an example showing how to create *excircles* for a given triangle, i.e. circles tangent to one side of a triangle and to the extensions of the other two sides.

```
> with(geometry):
point(A,-1,1): point(B,2,1): point(C,0,4):
triangle(T,[A,B,C]):
line(L1,[A,B]): line(L2,[A,C]): line(L3,[B,C]):
excircle (Circles,T):
draw({op(Circles),L1,L2,L3},printtext=true);
```



The geometry library is one of the most underestimated Maple libraries. The above example shows that it is definitely worthwhile exploring it.

WWW and Virtual Reality with Maple:

Nowadays, documents published on the Internet are part of the teaching resources that students can use for learning various disciplines. Web pages, in many situations are more convenient than printed documents. On them you can use a lot of colors without bothering about the cost of printing; you can use animations, and other elements illustrating mathematical concepts. In Maple you can save any Maple worksheet as an HTML file. Such a file after a few modifications can be placed on the web. Maple will save all graphs as GIF files and animations as animated GIFs.

Maple contains another option that is even more exciting than static web pages. With Maple you can create virtual worlds of our mathematical objects. Each plot can be saved as a VRML file and linked to a web page. Objects saved in VRML files can be manipulated by the use of a mouse pointer. You can zoom objects, rotate them, etc.

Final comments:

I have been teaching discrete mathematics for a very long time. I was able to model many topics in this discipline with Pascal or using Excel spreadsheets. However, there were always topics that were very difficult or even impossible to illustrate. After switching to Maple I found that I could quite easily demonstrate most of the hard topics with the use of simple Maple programs. Moreover, Maple is interactive and its programs do not require compiling. In Pascal, for example, compiling and debugging programs was a time consuming process. The flexible programming syntax in Maple gives students more freedom in expressing their ideas. In Pascal, even a small change of syntax produces a syntax error. Modeling operations on lists, sets or graphs in Pascal required introducing pointers and dynamic structures, which was quite difficult for first year students. In Maple I was able to work with sets, lists and graphs without bothering how to implement them. In consequence, tutorials with Maple were far more productive than similar tutorials with Pascal and Excel.

References:

Cheung C-K., Keough G. E., May Michael – *Getting Started with Maple*, John Wiley & Sons, Inc., New York, 1998.

Klimek G., Klimek M. – *Discovering Curves and Surfaces with Maple*, Springer, New York, 1997.

Developing Internet Resources for Online Teaching Mathematics with Scientific Notebook

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ABSTRACT:

In this paper I show how using a computer program called Scientific Notebook we can produce interactive material for online courses. I also point out the most important features of Scientific Notebook, which allow us to enrich this material and turn web sites into a valuable tool for online teaching of mathematics as well as developing online support for traditionally taught courses.I then describe what can be done to improve the integration of SNB documents with a web server. Finally I show just how useful can be SNB tests for online and online-supported teaching of mathematics.

Introduction

The idea of using the Internet for displaying mathematical concepts is not new. Since the beginning of the Internet educators and scientists have published a number of mathematical documents on the WWW. The major problem in this undertaking is the ability to display mathematical formulae on web pages. Documents on the WWW are coded in HTML, which is a kind of programming language for describing size and shape of characters, description of styles, tables and other components of a web page. HTML does not have the appropriate tags for coding mathematical formulae.

In recent years MathML, which is an extension of HTML, was developed. MathML contains all the possible tags for coding formulae. However, most existing browsers do not display MathML tags and future when these tags will be rendered at least by Netscape and Internet Explorer seem to be rather distant. Meanwhile many impatient developers of the WWW content started using pictures to display mathematical objects – formulae and diagrams (see [0, 0, 0]). This method is quite inefficient and time consuming. Imagine that a single page of mathematical text may contain hundreds of formulae. This means you need to maintain hundreds or thousands of pictures on the web site. Moreover, the reader of such a web page will have to download all these pictures to his computer. Imagine, how much time this could take? An example of such web site is the E. Weisstein's World of Mathematics website [0]. The development of this web site took about 8 years. You can also find a few other web sites on the Internet that use this method of publishing mathematics. Most of them were started long time ago and their authors have already given up updating the contents.

One of the most interesting methods of publishing mathematics on WWW was developed at Triad Computing Inc. (TCI). The key to this method is the replacement of HTML by TeX and LaTeX typesetting languages (see [0], [0]). TeX is a world standard for publishing mathematical and technical documents. LaTeX is one of its dialects. There are already millions of articles and books that were developed and electronically stored in TeX and LaTeX format. This gigantic library is ready to be displayed on the WWW if we accept TCI's solution.

TCI and recently MacKichan Software, has produced a word processor, known as Scientific Notebook (SNB), for developing LaTeX documents. SNB serves at the same as a web browser for LaTeX files. Additionally, SNB offers some other features that are not available in web browsers or in other programs designed for developing mathematical content. Describing these features is the major goal of this paper.

Dynamic versus static display of mathematics

HTML documents on the web are quite static. We can read them like a paper book. We can improve them by adding some Java applets and other active elements that will display animations of graphs and visualization of some concepts, however the major part will still be just a read-only document.

At the same time mathematics is very dynamic. While teaching mathematics we transform formulae, solve equations, plot graphs, etc. Imagine that we downloaded a mathematical web page. In order to change any formula on this page we have to develop it in a computer graphics program, save as a computer file and then embed it into our document. We would have to do the same with graphs, diagrams and other elements.

Now imagine quite a different situation. After downloading a mathematical web page, the student can recalculate each part of this document in the web browser. He can change coefficients of equations and explore alternative solutions. He can plot graphs of modified functions or animate them to see the shape of curves and surfaces better. Finally, he can take the whole document, add the solutions of the enclosed problems to it and post the final document to his instructor for grading.

Dynamic web documents, like what I described above, can be a very attractive alternative to the static web content. A number of computer programs can be used to develop such dynamic web content. You can use Maple V, Mathematica and Scientific Notebook. However, only SNB can satisfy mathematics educators, due to its attractive price and some other features that will be mentioned later. You can find more information about using SNB in teaching mathematics in [0, 0, 0, 0].

Why Scientific Notebook

People using telephone lines to browse Internet already know how important is the size of files that are downloaded by a web browser. Especially in online education, the size of files plays an important role. Students cannot afford to waste time while taking part in an online lecture or communicating with their instructor. This means that while developing web content we shall keep in mind the size of produced files and make them as small as possible.

SNB saves its documents in LaTeX, which is the most compact format of describing mathematics. LaTeX files are very small and may contain formulae, so we do not need hundreds of separate pictures for formulae. We also do not need to develop separate pictures with graphs.

Formulae in SNB documents are live. The secret of this feature is the Maple computing engine enclosed with SNB. Whenever we wish to calculate a formula in an SNB document, Maple V takes care of our formula, produces result and then this result is placed back in the document. At the same time SNB takes care of our interaction with Maple kernel. We do not need to know even a single Maple command to obtain results. At the moment SNB version 3.5 is distributed with two computing engines – Maple V R5.1 and MuPAD. MuPAD is a computer algebra system produced by group of scientists and students at the University of Paderborn in Germany. Many people believe that MuPAD computing kernel is even superior to those in some other CAS tools. In the future, it will be possible to type MuPAD code directly in SNB documents. This will give us an opportunity to trace algorithms in SNB and perform recursive operations programmed by users.

Scientific Notebook as an authoring tool

SNB contains two elements: the computing kernel and the word processor that acts as a user-friendly interface for this kernel. Besides of the communication with computing kernel this program is considered as one of the easiest to use mathematical word processors. Two its features are important.

Formulae in SNB documents are a part of the text and are not embedded elements like in most other word processors. This makes access to the formula quite easy. Just put the cursor onto it and type mathematics. You do not need to call an external formula editor as you do in MS Word and other programs.

In order to edit a formula, you do not need to click on hundreds of buttons and go through many dialog boxes. Just type it. There are a few important key sequences to insert roots, fractions, integrals, large operators, etc. For instance, in order to type $\sqrt{\frac{\sin x}{x}}$ just type [Ctrl][r] to obtain radical, then type [Ctrl][f] to obtain fraction and then type the rest of the formula. The formula editor in SNB had gained a lot of attention. The same formula editor was used in Ami Pro – one of the first Windows word processors – and later in Word Pro. At the moment the formula editor in Word2000 has implemented the same concept of editing a formula and the same key sequences.

In SNB documents, formulae are displayed in red to distinguish them from ordinary text.

Mathematical plots on the web

Graphs on the HTML pages are usually GIF or JPG files. Which means that their quality is not the best and files are large. When printing such documents, the quality of graphs is usually much worse.

In SNB web pages graphs are represented by formulae and drawn on the computer screen with the use of Maple or MuPAD computing kernel. Therefore, the graphs do not form separate files. SNB uses a vector format to display graphs on the screen, which doesn't lose its quality when we resize or squeeze the picture to extreme sizes.

Some more complicated graphs take a bit more time to be drawn on the screen. Usually this is still faster than downloading large GIF or JPG file. You may also consider of replacing live plots by their snapshots. You may ask SNB to save a snapshot of a plot and then embed it into document. SNB snapshots are produced as WMF files, which are vector files and their size and quality does not change with the size of the picture on the screen. SNB also supports many other graphical formats. However, you always have to keep in mind that the size of graphics files is very important.

I mentioned before that SNB documents are saved as TeX files. We can also save them as RAP files, where both text and embedded graphs will be wrapped together into one single file. This helps a lot in maintenance of the web site.

Mathematical quizzes on the web

We need to test our students in order to know how much they have learned and what kind of problems they encounter while learning the subject. Most known web based tests are single purpose multiple-choice tests. This means that every time a student accesses the test page he gets the same test. Such tests can be used only once and every time we are develop such a test we have to do all the work from scratch. This is very time consuming. However, it is not possible to do more intelligent tests without having a computing engine.

SNB has two good computing engines and very powerful test builder. Tests in SNB are not just multiple-choice tests. If you like, you can use multiple-choice design; however you may ask your student to directly write his response, and this response will be evaluated in SNB. In order to understand how SNB tests work, we have to go through the whole process, step by step.

STEP 1 – PROGRAM YOUR TEST QUESTIONS

Programming test files is quite logical. For each question you have to define variables for the question, write down question statement and define some choices. Here is an example of a programmed question:

Question
Setup
r:=5rand(2,19)
p:=5rand(10,39)
a:=100r/p
Conditions: $(r<p)\wedge((p\bmod 100)\neq 0)$
Statement
r is what percent of p?
Choices
- $a+5$
- $a+1$
- a $\boxed{\text{correctchoice}}$
- a-1

Fixed None of these.

Just reading the above code you may have noticed that variable r is a random integer taken from the interval {2, 3, 4, 5, … , 19} and p is a random number from {10, 11, …, 39}, thus a which is obtained from two random numbers will change its value depending on r and p respectively. Finally, given conditions reduce the choice of r and p to only specific situations.

The two randomly generated numbers r and p will be inserted into the statement that will be displayed to student. The next part, the choices, will display 5 possible solutions to the student. The choices will be placed in random order – there is a general instruction that makes it for all questions. The correct choice is a and this line will occur somewhere between other choices in random location.

STEP 2 - OUTPUT

Each time a student loads such test into his computer he will get different output. Here are shown three of them.

I 60 is what percent of 120? ○ 51 ○ 45 ○ 55 ○ 50 ○ None of these
I 40 is what percent of 90? ○ $\frac{400}{9}$ ○ $\frac{445}{9}$ ○ $\frac{355}{9}$ ○ $\frac{409}{9}$ ○ None of these
I 55 is what percent of 180? ○ $\frac{275}{9}$ ○ $\frac{320}{9}$ ○ $\frac{230}{9}$ ○ $\frac{284}{9}$ ○ None of these

STEP 3 – MARKING RESULTS

After choosing one of the possible choices and clicking on the button [Submit] that is placed at the end of the test document student will obtain his grades in the following form:

Student: John Brown
Started: Thu Apr 06 15:39:08 2000
Finished: Thu Apr 06 15:39:31 2000
Elapsed: 7 minutes 23 seconds

- Question 1: You selected choice 4 (5/7). The correct selection is choice 3 (343/20).
- Question 2: You selected choice 2 (82/2). The correct selection is choice 1 (75/2).
- Question 3: You selected choice 4 (1350). The correct selection is choice 3 (200/3).
- ....

You got 0 points out of a possible 10.

SNB tests (QIZ files) can be placed on the web site like other TeX or RAP files. This makes our tests accessible to students at any time. They can open them, as many times as they wish and work with them like with traditional books of problems. But there is one important difference – this problem book is dynamic and changes whenever a student opens it. Many educators use SNB tests as a drill tool for improving the students' knowledge of the subject, as well as their confidence in their knowledge.

Finally we have to think of how we collect the students' marks. For online education it is very important that the SNB exam builder can post results to a database on the web server. This way we can find out: how many times student attempted to do the test, how much time he spent doing test, which questions were the most difficult for him, his score for each attempt, etc. We do not need to spend time marking students work and assignments. We also gather much more information about our students than using traditional methods. This is especially important when we have large groups of students.

Designing mathematical pages

For about 500 years calligraphers and later printers learned how to design printed documents to grab as much as possible of the readers' attention. They learned a lot about the psychology of human reception. With the invention of computers these things still remain true. Only one thing has changed – web documents require more attention, as reading text on the computer screen is much more difficult than reading a newspaper or book. Thus designing issues for the web are even more important than those for the paper documents. Such things like the use of fonts, colors and grids must be very carefully considered.

Pages designed in HTML can be very colorful and appealing to readers. We can use various colors, banners, rulers, tables for formatting text and pictures, etc. It was always surprising for me that many mathematical web pages look rather ugly. Usually they have just white or gray background and black, often, ragged characters. The reason can be a lack of designing skills as well as the limitations forced by using pictures to represent formulae.

Very often, the size of the formulae is different than the size of the text. We must remember that in most web browsers, the user can change the size of characters displayed on his screen. At the same time pictures with formulae will still have the same size, thus making the displayed text very messy.

SNB documents look quite different. The whole text is formatted accordingly to the used style sheet. This makes the whole document very consistent. The concept of styles is exactly the same like in HTML. We deal with body text, header 1, header 2,..., lists and other elements that can be applied to the whole paragraphs. We may also define styles that we can apply locally: bold, italic, etc.

The user can define his own set of styles and use it for all his web site creations: both documents and tests. Creating such a style sheet is quite easy and it can be done while typing text. After finishing the document we have to also save our styles in a separate file; thus, we obtain a ready set of our favorite styles.

SNB comes with a number of professionally designed styles – for articles, books, lecture notes, online documents, etc. It is especially worth searching for good examples of styles in some online courses like [00], [0].

Other designing tools in SNB are similar to those that we have for traditional web pages. We can use tables for multicolumn formatting, banners and rulers, illustrations, etc.

Integration with web servers

Different web servers can deal with information published on the web site in slightly different ways. In a few cases you may find that web server as well as our browser doesn't recognize RAP and QIZ files. In this situation, the RAP or QIZ file is downloaded to the temporary Internet files on our computer and opened in the web browser window as a text file. One may expect that such file will be opened in SNB according to its type. This problem can be solved easily on the web server side.

You can ask your webmaster to find the file with definition of mime types on the web server. For example, for Apache and WebCT servers on a UNIX machine this is the file **mime.types** which is usually located on the directory:

/[installation directory]/webct/webct/server/conf/mime.types

The contents of the file **mime.types**, is a list of file types that shall be recognized by the web server. There are two columns. The left column contains the file type and the right column possible extensions of files of this type. Here you may find TeX related entries:

application/x-latex latex
application/x-tex tex
application/x-dvi dvi
application/x-texinfo texinfo texi

Note that there is a separate entry for TEX files but there are no entries for RAP and QIZ files. This explains why TeX files are handled properly by web server and SNB but RAP and QIZ not. So, using any text editor available on your server expand the TEX entry into:

application/x-tex tex rap qiz

This shall solve all your problems. Sometimes it is also worth to do some tuning of the web browser on the client side. This can be very important when you use links from HTML pages to an SNB document. Such modifications are especially important, on both server side as well as browser side, when we use a WebCT server for online teaching. You can find more detailed information about integration of SNB documents and WebCT in [0]. If you are developing a web site with interlinked HTML and SNB documents you may also find some important information in [0].

Conclusions

Education online means much more than only displaying documents on the web site. This also concerns teaching of mathematics online. However, perfect preparation of documents for the web is the first and very important step. All traditional methods are unsatisfactory and there are many reasons why they should be avoided. Using SNB for producing mathematical content for online teaching solves many problems with displaying mathematics online as well as adds a number of attractive features.
The three the most important features of SNB for online teaching mathematics are:

1. The ability of producing dynamic content for the web.
2. The ability of using formulae for coding graphs of functions and other mathematical documents rather than pictures in GIF or JPG format.
3. The ability of producing online tests, marking them and submitting scores to the database on the web server.

SNB gained already a lot of attention from mathematics educators. Many professors are using SNB as an electronic whiteboard for their lectures, while some others are producing electronic books on CDs or on the web (see [0], [0]). A number of large and small universities have decided to use SNB for their online education. Between them are the Texas A&M University (see [0, 0, 0]), the Concordia University, and the Hong Kong Open University.

In the last year, the Inter-University Institute of Macao started using WebCT and SNB for teaching Mathematics Education courses. These courses are taken stationary. However, a web site is used to support learning and to extend contact with teaching professors. By using a hit counter on the main page of each course I found that my students access lecture notes each night many times. Web site with

SNB documents became for them a source of information that they can access at any time.

References:

Allen D., Rahe M. – *Multimedia Calculus*, http://www.math.tamu.edu/~webcalc/ Texas A&M University, 1998.

Majewski M. - Evaluation of Scientific Notebook as a Tool in Mathematics Education, PNG *Journal of Mathematics Computing and Education*, Vol. 3 No. 1, pp. 25-38, 1997. Available also as an electronic publication at MacKichan Software, Internet address: *http://www.mackichan.com/techtalk/articles/articles.html*, 1997.

Majewski M. - How to Integrate Scientific Notebook documents with HTML files on a WWW site, electronic publication, TCI Resource Center, Brooks & Cole Publishing (now MacKichan Software), Internet address: *http://www.mackichan.com/techtalk/articles/articles.html*, 1997,

Majewski M. - *TCI-Web -- Method of Publishing Mathematics on the Internet, Electronic publication*, TCI Resource Center, Brooks & Cole Publishing (now MacKichan Software), Internet address: http://www.mackichan.com/techtalk/articles/articles.html, 1998.

Majewski M. - *Scientific Notebook - an Interactive Approach in Teaching Mathematics*, in Proceedings of the International Conference on the Teaching of Mathematics, John Wiley & Sons,1998.

Majewski M. - *Publishing Live Mathematics on the WWW, PNGJMCE* vol. 4, pp. 63-72, 1998.

Majewski M. - HTML Approach in Publishing Mathematics on the WWW, PNGJMCE vol. 4, pp. 53-63, 1998.

Majewski M. - Publishing Mathematics on the World Wide Web, Blackwell Science Oxford, *Journal of Computer Assisted Learning*, 1999, vol. 15, nr. 2, pp.139-148.

Majewski M. - *Experiences in Teaching Mathematics with Scientific Notebook, Mathematics for the 21st Century*, SEACME-8 conference proceedings, Manila, 1999, pp. 267-275.

Majewski M. - *A Survey on Methodology of Publishing Mathematics on the WWW*, (A summary of my previous papers and reports related to publishing mathematical concepts on the Internet), available at http://www.iium.edu.mo/mirek/publ_math/publ_maths.html,

Majewski M. - Teaching Mathematics with Scientific Notebook, The Challenge of Diversity - Symposium on Undergraduate Mathematics, Delta 99, pp. 118-125, 2000.

Majewski M. - *Why we need Scientific Notebook for Teaching Mathematics*, Mathematics 2000 Festival, University of Melbourne, 10-13 January 2000.

Majewski M. - How to integrate Scientific Notebook and WebCT server, *The Math/Science-Online Newsletter*, Texas A&M University, 2000.

Stecher M. – WebAlg – An on-line college algebra course, *The Math/Science-Online Newsletter*, Winter 1999/2000, Texas A&M University.

WebAlg Group – College Algebra, A Web Based, Interactive, Multimedia College Algebra, *http://www.academicsolutions.com/webalg/mindex.tex*

Weisstein E. – Eric Weisstein's World of Mathematics, at http://mathworld.wolfram.com/ , 2000.

Euler-type Formula Using Maple

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ABSTRACT:

Euler [1] represented $\varsigma(2)$ *as* $\varsigma(2)=3\sum_{1}^{\infty}\frac{1}{n^2\binom{2n}{n}}$. *In this article, we find a similar new representation for* $\varsigma(3)$ *in terms of* $\sum_{1}^{\infty}\frac{1}{n^3\binom{2n}{n}}$ *(correct to 9 places) using a previous paper of the author [2] and the Computer Algebra System Maple [3]. The importance of the result is also due to the fact that* $\sum_{1}^{\infty}\frac{1}{n^3\binom{2n}{n}}$ *converges more rapidly than* $\sum_{1}^{\infty}\frac{1}{n^3}=\varsigma(3)$ *which is still an unknown constant.*

AMS (1991) subject classification: Primary 40B05, 33E20, Secondary 11M99, 11Y99
Key words: Zeta of 3, binomial sums

In [2] the author found the following representation:

$$\varsigma(3)=\frac{\pi}{2}\sum_{1}^{\infty}\frac{\sin(\frac{n\pi}{3})}{n^2}-\frac{3}{4}\sum_{1}^{\infty}\frac{1}{n^3\binom{2n}{n}}.$$

Now

$$\sum_{1}^{\infty}\frac{\sin(\frac{n\pi}{3})}{n^2}=\frac{\sqrt{3}}{2}(1+\frac{1}{2^2}-\frac{1}{4^2}-\frac{1}{5^2}+\frac{1}{7^2}+\frac{1}{8^2}-...)$$

$$=\frac{\sqrt{3}}{2}\{-\frac{2\pi^2}{9}+\frac{1}{3}\psi'(\frac{1}{3})\} \qquad \text{by [4],}$$

where $\psi(z)=\frac{d\ln\Gamma(z)}{dz}$ (the logarithmic derivative of the Gamma function).

$$= -\gamma + \sum_1^\infty \left(\frac{1}{n} - \frac{1}{z+n-1}\right)$$ (where γ is Euler's constant, [6, p.259, 6.3.16]).

Thus, in particular,

$$\psi'\left(\frac{1}{3}\right) = 9\sum_1^\infty \frac{1}{(3n-2)^2},$$

and therefore

$$\varsigma(3) = -\frac{\sqrt{3}}{18}\pi^3 + \frac{3\sqrt{3}}{4}\pi\sum_1^\infty \frac{1}{(3n-2)^2} - \frac{3}{4}\sum_1^\infty \frac{1}{n^3\binom{2n}{n}}.$$

This result is new and enhances the previous result of the author [2].

At this juncture we first hoped to express $\sum_1^\infty \frac{1}{(3n-2)^2}$ in terms of π^2 only, given the shift by 2 of an already known series, which would give an amazingly pretty result. However, until now, this proved almost impossible and indeed to find the exact value of the above simply looking series is a difficult open problem known also as $\Psi(1,3)$.

Then, we turned to Maple [3] hoping to discover a numerical relation (which does not, in general of course, constitute a proof since the computer operates on rational approximations of numbers). To do so, we used "Integer relation algorithms" which are main tools for computer-assisted mathematics:

Definitions: Let $r \in \mathrm{R}^n$ be a given vector. We say that the vector $c \in \mathrm{Z}^n$ is an integer relation for r if $\sum_1^n c_k r_k = 0$ with at least one non-zero c_k.

An integer relation algorithm searches therefore for such a non-zero vector c_k.

Here we use the available LLL (Lenstra, Lenstra, Lovasz [5]) algorithm which is implemented in Maple and known there as the lattice-based relations algorithm. For this to be successful we must have some idea of the result sought.

```
>Digits:=12;

                Digits := 12

>a:= evalf(sum(1/(3*n-2)^2,n=1..infinity)); b:= evalf(ln(2/3)); c:= evalf(Pi); d:= evalf(Pi^2);

                a := 1.12173301393
```

b:=-4054651081081

c := 3.1415926535

d :=9.8696044010

```
> A:= trunc(10^10*a); B:= trunc(10^10*b); C:= trunc(10^10*c); D:= trunc(10^10*d);
```

A := 11217330139

B := -4054651081

C := 31415926535

D := 98696044010

```
> v1:= [A,1,0,0,0]; v2:= [B,0,1,0,0]; v3:= [C,0,0,1,0]; v4:= [D,0,0,0,1];
```

v1 := [11217330139, 1, 0, 0, 0]

v2 := [-4054651081, 0, 1, 0, 0]

v3 := [31415926535, 0, 0, 1, 0]

v4 := [98696044010, 0, 0, 0, 1]

```
> readlib(lattice):
> lattice([v1,v2,v3,v4], integer);
```

[[-95, 15, -140, -14, -3], [-329, -213, 332, -478, 190],

[147, -773, -169, -327, 185], [-476, -203, 124, 943, -272]]

\>

\>

Note that the point of the LLL algorithm is to find vectors whose components are small.

Thus we can write

$$15\sum_{1}^{\infty} \frac{1}{(3n-2)^2} - 140\ln\left(\frac{2}{3}\right) - 14\pi - 3\pi^2 = 0$$

Or

$$\sum_{1}^{\infty} \frac{1}{(3n-2)^2} = \frac{28}{3}\ln(\frac{2}{3}) + \frac{14}{15}\pi + \frac{1}{5}\pi.^2$$

Now we see that

$$\varsigma(3) = \frac{17\sqrt{3}}{180}\pi^3 + \frac{7\sqrt{3}}{10}\pi^2 + \left(7\sqrt{3}\ln(\frac{2}{3})\right)\pi - \frac{3}{4}\sum_{1}^{\infty}\frac{1}{n^3\binom{2n}{n}}.$$

We now test the above result using Maple to see how well the algorithm worked .

```
>Digits:=12;

                    Digits:=12

>evalf(17*sqrt(3)*Pi^3/180+7*sqrt(3)*Pi^2/10+7*sqrt(3)*ln(2/3)*Pi-
3/4*sum(1/((n^3)*binomial(2*n,n)),n=1..infinity),12);

                    1.20205690570

>evalf(Zeta(3),12);

                    1.20205690316
```

Consequently, we see that the above representation we gave for $\varsigma(3)$ is correct to 9 places.

Concluding remarks and future project. The following two formulas are easy to prove [9]:

There is an interesting identity due to comtet [4] that

$$\sum_{1}^{\infty}\frac{1}{n^4\binom{2n}{n}} = \frac{17\pi^4}{3240}, \quad \sum_{1}^{\infty}\frac{1}{n^2\binom{2n}{n}} = \frac{\pi^2}{18}.$$

But there are no known values for

$$\sum_{1}^{\infty}\frac{1}{n^k\binom{2n}{n}}, \quad k > 4.$$

In 1979, Apery [7,8] amazingly proved that $\varsigma(3)$ is irrational but it remains open whether

$$\frac{\varsigma(3)}{\pi^3}$$

is irrational. In any case, and based on the above remarks, it seems like the ideal project (though very difficult, and with or without the help of the computer as a tool to come up with a claim which we will then prove mathematically) is to try to look for an identity for $\varsigma(3)$ solely in terms of

References

[1] David H. Bailey, Jonathan M. Borwein, Peter B. Borwein, and Simon Plouffe, "*The Quest for Pi*", Mathematical Intelligencer, 19 (1997), 50-57.

[2] Badih Ghusayni, "Some representations of $\varsigma(3)$ ", *Missouri Journal of Mathematical Sciences*, 10 (1998), 169-175.

[3] B. W. Char, K. O. Geddes, G. H. Gonnet, B. L. Leong, M. B. Monagan, S. M. Watt, *Maple V Language Reference Manual*, Springer-Verlag, New York, 1991.

[4] Gunter Bach, "$\varsigma(n)$, $\psi^{(n)}$, and an Infinite Series", *American Mathematical Monthly*, 81(1974), 180-181.

[5] A. K. Lenstra, H. W. Lenstra Jr., L. Lovasz, "*Factoring polynomials with rational coefficients*", Math. Ann. 261(1982), 515-534.

[6] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions*, Dover, New York, 1972.

[7] R. Apery, "Irrationalite de $\varsigma(2)$ et $\varsigma(3)$ ", *Journées Arithmetiques de Luminy, Asterisque*, 61(1979), 11-13.

[8] A. van der Poorten, *A proof that Euler missed...Apery's proof of the irrationality of* $\varsigma(3)$,Math Intelligencer 1(1979), 196-203.

[9] Jonathan Borwein and Peter Borwein, Pi and the AGM, Wiley-Interscience, New York, 1987.

[10] L. Comtet, *Advanced Combinatorics*, Dreidel, Dordrecht, 1974.

WWW Mathematics at the University of Pretoria

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ABSTRACT

At the University of Pretoria we embarked on designing a web based course for first year Calculus students. This was done within a WebCT framework. The course ran on an experimental basis during the first semester of 2000. This paper discusses the structure of the course and the problems encountered on the way. We also discuss feedback received from students as well as plans for the future.

Background

It was a matter of time before we jumped on the bandwagon. At the University of Pretoria the Telematic Education Section encourages and offers excellent support for courses that run on the web. Our aim at the Mathematics Department was to offer a first year Calculus course on the web.

Our web-based Calculus course has a specific target market. At this university we have something called anti-semester courses. Students typically enroll for their first Calculus course at the start of the first semester. If successful they continue with a follow-up course. If not, they can repeat the exact course in the second semester with the benefit of not having to skip a semester. This course is then called an anti-semester course. Although the majority of students enter the course as repeaters they are joined by a fair number of first-timers. The majority of anti-semester students seem to be reluctant to attend class, they've heard it all before. They simply need more time to work through the course matter again. With this reasoning in mind it seemed obvious that anti-semester courses should go on the web.

Taking the First Step

As the Telematic Section supports *WebCT* the decision was made for us to follow this route. *WebCT* is a web environment suitable for accommodating university courses. We started by attending a *WebCT* course presented by the Telematic section [Drysdale: 1999]. This is advisable as an introduction to the *WebCT* world and offers the big picture. It also included a session on using Front Page for creating HTML pages necessary for the uninitiated. As with any other course it is impossible to become an immediate expert and plenty of the info is soon forgotten but the basic structure remains in your head and it is then simple enough to find your way to whatever you need.

The Academic Approach:

A very important decision was on the amount of subject content to be included in the actual web pages. There are clearly a number of different options that one can follow.

Many universities use the internet to provide a variety of mathematical problems and/or demonstrations [Allen et al: 1998]. Others use computer algebra systems in their courses [Back]. The sites [Davis et al] and [Uhl & Woods] are probably some of the most intensively administered and comprehensive sites in existence. They use *Mathematica* notebooks for their courses.

Eventually we decided that we will not try to rewrite the textbook on the website – our approach is that for the actual subject content, the student will still have to use the textbook. In the web course we try to guide the student through the subject content by providing

- Detailed learning objectives for every section
- Guiding notes linking the textbook to the learning objectives
- Dynamical day by day scheduling as a pace setter
- Prescribed problems, also scheduled
- Complete and/or partial solutions to the problems (after a time)
- Other relevant information about the course, such as information about assessment, etc.

A second important decision was on assessment. Logistically we cannot provide for students writing their semester tests and examinations away from campus and we had to force students to do these on campus.

Structuring the Web Page:

Having established the academic approach, we then had to structure the web pages. We looked at the following main divides.

Content

The *WebCT* environment for a web-based course consists basically of two parts, the first being the more permanent section that you rarely update and the dynamical section which is updated daily. In the more permanent section we house the study guide with detailed learning objectives and references to the applicable section in the textbook. The study guide is divided into Themes, divided into Study Units. For each Study Unit the learning objectives are listed followed by certain remarks (telling the student what to omit in the textbook etc) and the selected problems. The Study Guide in packaged under CONTENT. The telematic section was quite willing to undertake this fairly voluminous task of getting the Study Guide on the Web and they prefer us not to meddle with it unnecessarily.

Also in the more permanent section is the FAQ (Frequently Asked Questions) which we were advised only to update at the end of a semester, particulars of the LECTURER(S), how we plan to do ASSESSMENT and an excellent HELP function.

Lecture Notes

One of the more difficult decisions was on the short lecture notes that we planned to present daily. The plan was to present a shorter version of what you would normally do in class, that is an interpretation of the textbook. The question now was *where* to put this, *how* do we do it and *what* should these lectures consist of?

For the *where* we decided on *Web CT's* BULLETIN BOARD. Here we post the daily lectures, each theme in a different FORUM. Hence in the forum called NOTES FOR THEME 2 you find notes on all the Study Units in this theme. We also created forums for the solutions of problems in each theme.

The *how* question was more difficult to answer. To put mathematical symbols on the web is notoriously ridden with difficulties. We also realised right at the start that although many staff members were *Latex* users it was not feasible to expect student assistants and typists to master it easily (or for us to teach them). After reading widely we decided on *Scientific Workplace* which really is a front end to *Latex*. *Scientific Workplace* is extremely simple to master and has the added benefit of being able to read *Latex* files. So we decided on *Scientific Workplace* as a facility to write the daily notes as well as the solutions. It would be a folly to say that the road from *Scientific Workplace* to the web was an easy one for us to travel. Interesting and simple once you look back, but not easy if you travel it blind the first time.

The *what* question was equally difficult. We definitely did not want to repeat everything in the textbook. On the other hand, our experience is that students really struggle to read a textbook without any guidance from a lecturer. Even at this moment we are still not certain that what we are including in the Notes is what the students really need. Our approach is to try to interpret the content of the textbook and to act as a go between the study guide, containing the learning objectives, and the textbook, containing the actual subject content.

Scheduling

For putting the whole course together we make use of the SCHEDULE facility (or diary) and this is where the student departs from. He opens up the SCHEDULE and goes to the particular date. Under a heading *What happens today?* we name the study unit of the day and then link it to the short lecture on the BULLETIN BOARD and the objectives under CONTENT. The SCHEDULE is also the ideal place to announce forthcoming events such as semester exams etc.

Running the course

Being inexperienced, we decided to run our web course for a trial semester, giving access to all our mainline Calculus first year students (all 1000 of them). This has proven to be a wise step because there are definite hiccups to sort out and this period of grace was invaluable.

Technical details

As mentioned before we decided on using *Scientific Workplace* which offers a front end to *Latex*. On screen one sees a WYSIWIG version of the *.tex file behind it. It is a *Windows* program with all symbols given as icons and it is extremely simple to use. Convenience always has a price to pay and it is true that *Scientific Workplace* doesn't offer the same flexibility as does *Latex* but for lecture notes and solutions it proved to be far more than adequate.

Having decided *on Scientific Workplace* for typing up notes and solutions (and acquiring a number of copies of the programme at quite a cost) the next step was to find the way of transferring the *.tex files onto the web. There are a number of tex to html converters in use such as *TtH* and *Tex2Html*. These can be downloaded and seem to do the job reasonably well but not without hiccups.

- *TtH* ia a DOS-based programme that you let loose on a tex file to produce a html file. As long as you use DISPLAYSTYLE math it works fine but it does not like in-line expressions at all. The end product is okay but definitely not of the same quality as a latex document. [Hutchinson: 1999]
- *Tex2HTML* is another option. It makes use of little GIF pictures for producing mathematical symbols that makes for a larger file.
- A third option would be to use *Scientific Workplace* or *Scientific Notebook* as browser. This has the advantage that the *.tex files can be transferred directly to the website. In this case the reader needs a copy of Scientific Notebook or at least the Scientific Viewer to read the *.tex files. [Majewski : 1997],[Lewin : 1998]

What we eventually decided upon was to convert the *Scientific Workplace* files to PDF (Portable Document Format) files. The quality is excellent, students can easily make a printout and the on-screen the file can be enlarged. For reading the file on the receiving end it is necessary to have *Adobe Acrobat Reader*. This can be downloaded from the web at no cost if not in use already. From the sending side one needs *Adobe Acrobat Distiller* (Version 4 is available now) that has to be purchased and any postscript writer.

So the path we follow is: Create a *Scientific Workplace* (*.tex) file, then a Post Script file (*.prn) and then a use *Acrobat Distiller* to create a PDF file (*.pdf).

The reasoning behind converting to a Post Script file first, is that a PDF file is too clever for its own good. It carries along a lot of info regarding the structure and fonts of the file. Postscript is "less intelligent" and specifies what goes where without carrying the font along. By then applying the DISTILLER a PDF file is created but still without the offending font

When posting the file (to the Bulletin Board of *WebCT* in our case) the PDF-file is attached. When the student goes to the appropriate place on the Bulletin Board, the *Reader* is automatically called to open the PDF-file. We were ecstatic on discovering the simplicity and beauty of this process.

Feedback:

Although our first run was only a trial run, we obtained some feedback from the students by conducting two surveys. From the student records of *WebCT*, we could extract the names of all students that visited the site more than 10 times through the semester. They were asked to complete a questionnaire viewing their experiences on the course.

Accessing the web course was completely voluntary. Students were informed - inspired in some cases (depending on the enthusiasm of the particular lecturer about the course) - about the existence of the course and then it was up to the individual to make use of it or not. Normal lectures carried on as usual and the web course was an extra.

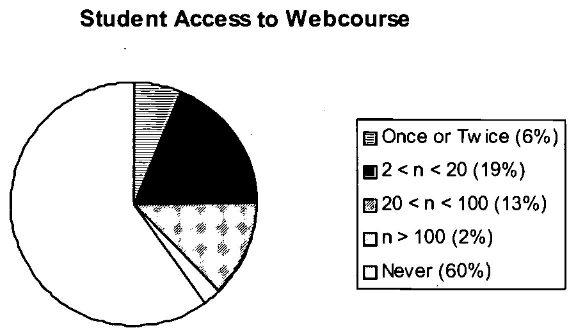


Figure 1

All students attended normal lectures and could voluntarily be part of the web-based presentation. The majority of the students were not part of it at all, as indicated by Figure 1.

We compared the semester marks of this group (of 113 students) with the entire group (of 930 students). In Figure 2 the percentage of students with semester marks in the given intervals are given.

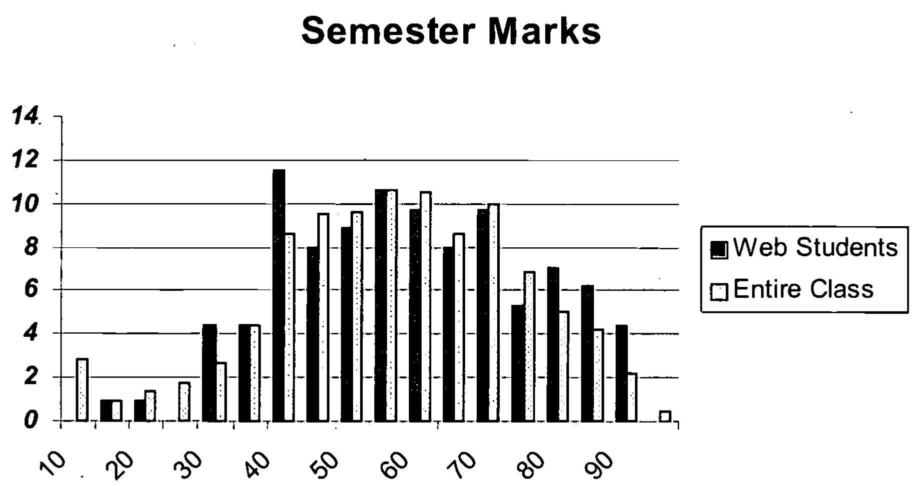
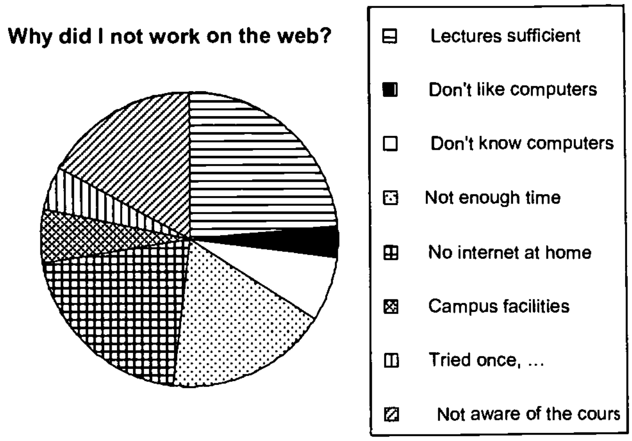


Figure 2

There is no significant difference between the web students and the entire group, although it does look as if percentage-wise, there are more web students than other students in the high scoring intervals 80-85, 85-90 and 90-95. An interesting feature is the peak in the web student group for the 40-45 group. These students are obviously the ones in a crisis who would try anything that might lead to salvation.

We also asked the other students why they did not make use of the web and the answers here were quite interesting.

BEST COPY AVAILABLE



Why did I not work on the web?

Figure 3

From Figure 3 it is clear that our students are still largely in the lecture frame of mind and still have to adjust to the idea of a different teaching mode. A large group considers lectures to be sufficient. Another significant group did not have internet facilities at home or found the university facilities insufficient. This stands to reason in the South African context where internet facilities at home are only now becoming popular and universities are still in the process of providing sufficient facilities.

An interesting question was which section of the site students read most frequently and from Figure 4 it is clear that the daily notes we wrote and the solutions to the assignments were the most popular.

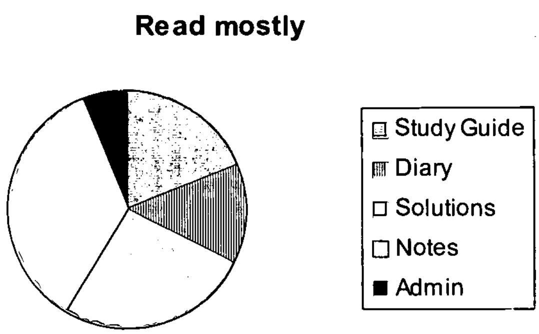


Figure 4

Students were asked whether they would be able to cope with the course without any lectures and working only on the web course. The results, given in Figure 5, were not too positive but should be seen in context. Very few (only about 5 students) worked on the web course really intensively, in the sense that they visited the site on an almost daily basis. Furthermore, students are used (and spoiled) to formal lectures. This response was therefore expected.

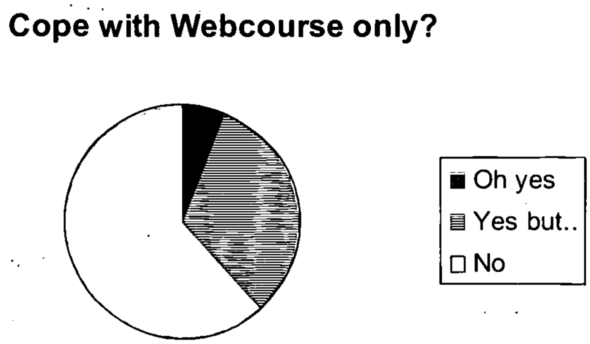


Figure 5

In personal interviews with some of the web students as well as in written comments on the course, we had a lot of very constructive feedback. Some of the recommendations will certainly be built into the course for the next semester.

Student Comments

Constructive comments were that instructions for accessing and navigating the web course should be included in the study guide. They also suggested a user-friendlier interface to eliminate what some of them considered "confusing navigation". As for the notes, a very important part of the web course, students wanted more worked examples that might be considered as a reflection on the textbook. They also wanted online tests (for self-testing), more complete solutions to problems and (students will be students) plenty past exam papers. Other requests were for a chat room and a search engine.

On the negative side, students did not appreciate the necessity of a password. They also thought that the web course was poorly advertised - a factor that can be directly attributed to lecturers who were uninvolved or disinterested. Then there was the matter of bilingualism, a fact of life in South Africa. At the University of Pretoria there are many students with Afrikaans as first language and students have a choice as whether to attend lectures in Afrikaans or English. We did not feel that we could afford to do the course in two languages and did not accommodate Afrikaans students in the same way on the web.

More positive comments came from students that felt that we should at all cost carry on with developing the web course. They liked the informal style, valued the lecture notes highly and (of course) appreciated the solutions to problems that were presented on the web. The web course even earned the typical student complimentary word of "cool".

Conclusion

From the student comments and our own experience we see the need of easier navigation on the site and also of scheduled chat room sessions. We have also decided to include regular self-testing and more worked examples. We plan to extend the daily lecture notes and to brighten this up with the use of more color and graphics. Due to popular demand we will assign fewer problems and provide more solutions.

Designing and running our web-based Calculus course has been a tremendous learning curve for us and we are appreciative of the enthusiasm with which many students received it. The web is here to stay and it is our task to harness it for our use as a learning tool.

References

Allen D, M Rahe, M Stecher, P Yasskin (1998) *A Web Based, Interactive Multimedia Calculus*, Texas A&M University, Electronic Publication at http://www.math.tamu.edu/~webcalc/

Back A *Math Department Instructional Computing Lab: Multivariate Calculus,* Cornell University, Electronic Web Course at http://mathlab.cit.cornell.edu/local_maple/mvc/lecguide.html

Davis W, H Porta, J J Uhl *Calculus & Mathematica,* University of Illinois, Ohio State University, Electronic Web Course at http://www-cm.math.uiuc.edu

Drysdale E (1999) *Guidelines for the Development of Web-based Courses for the Virtual Campus* Publication of the University of Pretoria.

Hutchinson I (1999) *Approaches to WWW Mathematics Documents* Electronic Publication at http://hutchinson.belmont.ma.us/tth/

Majewski M (1997) *How to integrate Scientific Notebook documents with HTML files on a WWW Site.* Electronic Publication at http://www.mackichan.com/techtalk/articles/mmaj02.html

Lewin J (1998) *Placing Scientific WorkPlace or Notebook Documents on Websites* Electronic Publication at http://www.tcisoft.com/techtalk/teaching.htm

Uhl J J, D Woods *Netmath: A Distance Education Program,* University of Illinois, Electronic Web Course at http://netmath.math.uiuc.edu

An Alternative Sequence for the Calculus

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Introduction:

The Mathematics Department at California State University, Bakersfield (CSUB) has redesigned the calculus sequence for science, mathematics and engineering students, with the purpose of achieving the following objectives:

- Capture the best of both the traditional and reform treatments of calculus.
- Deal effectively and creatively with the issue of technology in the calculus.
- Introduce students to an exciting learning experience beyond the traditional, while ensuring that the expectations of formal computations are met (by formal computations, we mean computations without the use of technology).
- Have students gain facility and confidence with the use of computer algebra systems.
- Ensure a smooth and logical transition for the student from one calculus course to the next, and from the calculus sequence to engineering and the applied sciences.
- Increase retention rates through the calculus sequence without simplifying course content.

This paper contains three sections. In the first section, we describe the reasons which led us to make a change in our traditional sequence. In the second section, we describe the alternative sequence in detail and its implementation. In the third section, we discuss our observations about the alternative sequence after two years of implementation, and discuss possible modes of assessing the alternative sequence. This is strictly a viewpoint paper. Although we include a section on observations, the alternative sequence is in its infancy, so we do not have sufficient data to justify any judgment of our changes. The purpose of the paper is to describe our experiences, in the hope of helping any department who is considering implementing changes similar to the ones we have made. Any views voiced in this paper are the author's, and are not intended to represent CSUB's Mathematics Department as a whole.

Impetus fir Change:

The Mathematics Department at California State University, Bakersfield has begun the implementation of an alternative calculus sequence for science, mathematics and engineering students, which captures the positive features of both the formal computational approach, and a more modern approach which uses a computer algebra system (CAS) as a tool to enhance conceptual understanding. The alternative sequence includes two new courses which are laboratory in nature, where students work in groups on modules with the aid of a CAS.

Before the alternative calculus sequence was created, all mathematics courses at CSUB required for the major were 5 units, meeting 3 times a week for lecture, and once for a laboratory session, running for 140 minutes. Except for calculus courses, this set-up remains. Instructors have used lab sessions in different ways depending on the instructor and the course: During lab sessions, some instructors introduce software packages, some assign supplementary problems to be worked in groups and handed in, and some use the lab session for student presentations of assigned problems. In particular, for the calculus sequence, some instructors used the lab session to introduce students to technology, and some did not. Finally, although an Engineering Program is projected at CSUB soon, none exists today; but we do have a Pre-Engineering Program which feeds into neighboring programs. Students in the Pre-Engineering Program, as well as students in Mathematics, Physics, Chemistry, Computer Science, and Pre-Med are required to take (at least part of) the calculus sequence.

Rationale:

What should compose the content of a calculus sequence for science, mathematics and engineering students, and what is the best way to teach such a sequence? This simple question has been the source of much controversy in Mathematics Departments across the United States, and is part of a broad debate on the reform of college and pre-college mathematics. This debate is linked to a large extent to technology. The burgeoning of mathematical software over the last decade has given the mathematician the power to perform complicated computations, display intricate graphs, and solve multi-step problems in a minuscule fraction of the time it would take the mathematician to achieve the same result by hand." The retrieval of mathematical information becomes a matter of a few strokes on the keys of a computer. Retrieval of information, however, must not be mistaken for the imparting of knowledge, which is the educator's goal. The extent to which, and the methods by which, new modes of information retrieval should be used in mathematics education are questions of controversy today amongst mathematicians world-wide. We refer to this controversy as the technology debate.

The technology debate is very much alive at California State University, Bakersfield (CSUB). Our department has been experimenting for several years with various ways of incorporating technology into our calculus sequence. We have tried several books ([4], [5], [6]), and incorporated technology from the standard graphing calculator to Maple V (a computer algebra system) at all levels of the four quarter sequence and in varying degrees. This experimentation as a whole produced mixed results. On the positive side, it made members of the department more aware of, and more knowledgeable about the strengths and weaknesses of the reform ideas that were tried; on the negative side, it rid the sequence of the uniformity in content expectations that it once had. Indeed, members of our department, not unlike most mathematics departments nationally, had views on the technology debate which varied from no reliance on technology, to total incorporation of technology and thus radical change. While some instructors forbade the use of a CAS on homework assignments and in-class examinations and never used a CAS in the classroom, others only used technology in lab sessions, but not on exams, and yet others used technology in lab sessions, on exams, and in lecture, and so in the author's view, almost totally eliminating the possibility of their students acquiring formal computational skills. As a consequence, students who went through the calculus sequence toggling from one instructor with ideas on one side of the reform spectrum, to another with views on the opposite side, in our opinion, were put at a disadvantage. Indeed, to such students, what is expected in solving a calculus problem became subjective. For example, suppose that a student needs to determine whether the series $a_n = \sum^{n} 1/k$ converges or diverges. Is it enough (will the student get full credit) that the student plot a sequence of partial sums and note that they do not level off? If not, and if the student chooses to use the Integral Test to justify divergence, may the student use a CAS to solve the integral? Since the integral is improper, may the student use a CAS to compute the resulting limit? To use the integral test, the student must verify that $f(x) = 1/x$ is monotone. If the student chooses to prove monotonicity by computing $f'(x)$, may the student resort to a CAS? Finally, if a student does not have access to a CAS, should the student be able to perform the above computations formally? In the author's opinion, mathematics at the stage of university calculus should be progressive, with close to uniform expectations throughout the sequence. So how does a Mathematics Department today agree on a common set of answers to the questions above, while preparing the student well in the discipline?

Oblinger and Rush [3] concisely argued the following rationale for change in the traditional mode of instruction in the introductory sciences: The introductory science courses at many of our large universities around the world can be an intimidating experience for the new student. It is not only the difficulty of the material, but also the experience of sitting in the large non-interactive lectures with an instructor who is mathematically unapproachable even when personally approachable. " Even though we agree with the above assessment, we do not favor one side of the technology debate over the other. The principle which the author accepts is this: There is a value to the conception that students in calculus should be exposed to, and assessed on algebraic manipulations and formal computations, without the use of technology, as this reinforces conceptual understanding; there is equal value to the belief that since today's computer algebra systems enhance the student's conceptual grasp, and facilitate introducing the student to enticing mathematics beyond the calculus, it is consequently the case that computer algebra systems must be integrated into the calculus.

We propose a model for calculus instruction, which we believe, is a reasonable answer to the technology debate.

Implementation Description of the Alternative Sequence:

To implement the above principle without jeopardizing the uniformity of course content from one quarter to another, and while maintaining a smooth transition for the student from one course in the sequence to the next, the Department of Mathematics at CSUB has redesigned its calculus sequence. We describe the alternative sequence:

(1) The sequence is composed of the following quarter courses: Math 211 (Calculus I, 5 units), Math 212 (Calculus II, 3 units), Math 213 (Calculus III, 4 units), Math 214 (Calculus IV, 4 units), Math 222 (Laboratory Experience I, 3 units) and Math 223 (Laboratory Experience II, 3 units).

(2) Math 211 and 212 cover differential and integral single variable calculus and emphasize theory, formal computations without the use of CAS, graphical and algebraic relationships and problem solving through applications. Only calculators not equipped with CAS capability are allowed on exams. The text we are using for these courses is Stewart's *Calculus, Concepts and Context.*

(3) Math 222 aand 223 are lab courses, with heavy use of CAS (Maple V in our case), and consist of 8 to 12 modules each (depending on module length and choice of instructor). For each course, the student will attend two lab sessions a week, each session running 2 hours and twenty minutes. During these sessions, the student will use a CAS to reinforce concepts in calculus and will be introduced to areas of mathematics beyond the calculus. Math 222 is taken concurrently with Math 212. Math 222 modules primarily consist of material relevant to that in Math 211 and 212. Math 223 is taken either with Math 213, or with or prior to Math 214. Math 223 modules contain multivariable calculus, sequences and series and areas of mathematics beyond the calculus.

(4) Math 213 and 214 cover multivariable calculus and sequences and series and emphasize theory, formal computations without the use of a CAS, graphical and algebraic relationships and problem solving through applications. Only calculators not equipped with CAS capability are allowed on exams; however, instructors are encouraged to assign projects involving the use of a CAS, to which students have already been exposed.

(5) All courses have an upper bound of 30 on the number of students registered for the course.

A list of modules for Math 222 and Math 223 might be as follows:

<u>MATH 222:</u>

1) Introduction to Maple V and Review of Differential Calculus. (1 session)

2) Newton's Method for Finding Roots. (2 sessions)

3) Derivative Estimation. (2 sessions)

4) Numerical Integration. (3 sessions)

5) Average Value of a function. (2 sessions)

6) Applications of the Integral. (2 sessions)

7) The Gamma Function. (2 sessions)

8) Polar Coordinates. (2 sessions)

9) Area in Polar Coordinates. (2 sessions)

<u>MATH 223:</u>

10) Elementary Differential Equations. (2 sessions)

11) Modeling with derivatives - Population Dynamics. (2 sessions)

12) Taylor Polynomials and Applications. (2 sessions)

13) Applications of Taylor and Maclaurin series. (3 sessions)

14) Parametrizations of Curves. (3 sessions)

15) Projectile Motion - Does the Green Monster Desreve Its Name? (2 sessions)

16) Lagrange Multipliers and Linear Programming. (3 sessions)

17) Applications of Multiple Integration. (2 sessions)

The novelty in our program lies therefore in the introduction of the two laboratory courses and in the simultaneous inclusion of both traditional and non-traditional methods of teaching the calculus.

Robert Menges [2] argues that when subject matter is available via interactive technologies, there is less burden on the teacher to present information and more opportunity to diagnose learning problems and help learners find solutions. Research shows that when students work with computers, teachers reduce the time they spend directing students; they spend more of their time facilitating student learning." The aim of our laboratory courses is three-fold: First, to create self-learners through group and individual activities with a faculty facilitator; second, to provide students with every resource available to them in mathematics and specifically, to have students gain facility and confidence with a CAS, consequently strengthening their grasp of content; and third, to inspire students to continue with mathematics beyond the Calculus. The topics covered in these modules are those which we view as better suited for introduction via CAS. These topics include, but are not limited to: Elementary differential equations, population growth, mathematical modeling, integral and derivative estimation, application of single and multiple integration in the sciences, projectile motion, Lagrange multiplyers and linear programming, parametrizations, polar coordinates, Taylor polynomials, Taylor series, and various standard calculus algorithms such as Newton's method, the bissection method and Simpson's rule. The following example, taken from our module on polar coordinates (designed for MATH 222), illustrates the benefit of a CAS in developing intuition on a topic which otherwise would require tedious computations. Note that after using the CAS for discovery, the student is still required to justify findings using mathematical rigor.

Assignment 8.4 Polar curves obtained from equations of the form $r = a+b\sin(\theta)$ or $r = a+b\cos(\theta)$ are called *limacons*. There are four possible types of limacons: (1) cardioids, (2) limacons with inner loop, (3) dimpled limacons, and (4) convex limacons. The type of limacon one obtains depends on the quotient a/b. In particular the four cases $a/b < 1$, $a/b = 1$, $1 < a/b < 2$, and $a/b > 2$ produce the four possible types of limacons listed above.

a) Determine which case for a/b gives which type of limacon. Experiment with plotting $r = a+b\sin(\theta)$ for various values of a and b, then match the condition on a/b with the type of limacon. Give a sample function and plot for each type of limacon.

b) Justify algebraically why a limacon with inner loop has an inner loop.

Grading of the laboratory courses is largely, if not entirely, dependent on the student's laboratory report. In case group reports are allowed, some individual examination may be necessary to ensure that every student is participating in the learning experience.

Project Execution:

Before the alternative sequence began, several steps were taken to ensure smooth implementation. These steps are likely to be necessary in every department willing to adopt a change similar to ours, so we mention them here:

- Securing the support of all departments on campus which our calculus sequence serves. In addition to the Mathematics Department, these are: Computer Science, Chemistry, Liberal Studies with a Mathematics concentration, Geology, Physics, Pre-Engineering, and Pre-Med.
- Securing administrative support. This is essential for securing and maintaining a computer lab for the courses requiring CAS use. Also, in our case, the faculty effort nvolved in the initial design of the modules was supported with 11 units of assigned time awarded to the Mathematics Department by the administration. Of course, generally, a department can use one or several of many sources of calculus modules in the literature, instead of designing its own materials.
- Informing local community colleges and other affected programs of our changes and encouraging them to institute similar changes, offering faculty help, if needed.
- Securing a well maintained computer laboratory with easy access, reasonable hours of operation and sufficient machines.

Observation and Evaluation:

The implementation of the alternative sequence began in Fall Quarter 1998. The first offerings of Math 222 and Math 223 were Winter Quarter 1999 and Spring Quarter 1999 respectively, and these courses are offered every quarter. We therefore have had occasion to test the alternative modules. We describe our observations of the alternative sequence as a whole. On the positive side:

- Due to the laboratory courses, students get a strong handle on a specific CAS. This gives the student a forum in which solid mathematical intuition can be developed. We have found that many students often go to a machine, unprompted, to get a sense of a problem at hand.
- Students do not rely on a CAS to solve every mathematical problem. Since part of the alternative sequence is still taught stressing formal mathematics, the need for justifying assertions in mathematics is made clear and expected of the student.
- Students get in depth exposure to new types of calculus problems such as estimation, error analysis, and modeling of physical phenomena. Although important, such topics were previously omitted or barely treated by the instructor, largely due to limited time.

- Students get a flavor of topics beyond the calculus such as probability distributions, mathematical induction and linear programming.

- Students concentrate more on *writing* mathematics. The modules always require a written discussion of every investigation the student makes.

- Students learn to challenge results produced by the CAS, ending the common naive belief that anything a computer outputs represents truth. This empowers the student to defend their own conjectures with mathematical reasoning.

On the negative side:

- Most modules we designed are too long and elaborate. Indeed, it is easy to overshoot on the initial try due to the designers' desire to cover a topic well.

- The laboratory courses are unpopular amongst faculty for several reasons: Laboratory courses at our institution carry fewer units per contact hours than lecture courses. This, coupled with the facts that the course carries a high grading load, and that many instructors like to design their own modules, makes the time invested into the course by the instructor disproportionate to the number of units earned. Naturally, we are in favor of allowing instructors the freedom of designing their own modules as long as certain topics are covered in the course, to make sure that as much as possible of the full curriculum is presented. Having different instructors contribute modules allows each instructor to inject his or her own interest and individuality, and adds to the databank of modules.

- The number of modules already designed is sufficient for one offering of each laboratory course, but more modules need to be designed to prevent academic dishonesty issues from arising. Specifically, if the instructor cannot vary the modules in the course, then old write ups will eventually start circulating amongst students. A need for a databank of modules is therefore essential.

- The modules do not yet have as many modern applications as we would like. With time for research and writing, we anticipate the design of modules which tap into current data in the applied sciences (such as hydrology or physical chemistry), then using calculus to model system behaviour reflecting the data.

Overcoming the above points will require resources which the university must be ready to offer in time and money. In the words of P. Breivik [1], sooner or later every successful initiative will expand, creating a need for more resources -both people and dollars. While some unique aspects of the resource issue relate to the growth of information literacy programs, such growth -as with all other major new curricular efforts- will require creative reallocations and perhaps additional resources."

Since the alternative sequence is still in its infancy, the observations mentioned so far are not based on a quantitative study. In the long run, sufficient data needs to be collected to evaluate the program. Specifically, data needs to be gathered on:

- Retention rates through the Calculus sequence, and the rate of calculus students feeding into disciplines requiring calculus for the major. Data on these rates can be retrieved for the past ten years. Our goal is that the calculus sequence redesign to increase these rates.

- The performance of calculus students in courses with calculus as a prerequisite. Again, such data can be compared to (1) past students who fit the same category, and (2) students in the subsequent courses who have taken a calculus sequence off campus, which does not include a laboratory experience.

References:

Patricia S. Breivik *Student Learning in the Information Age*, The Oryx Press, 1998.
Robert Menges, book chapter in *Teaching Tips*, by Wilbert McKeachie, Heath and Company, 1994.
Diana G. Oblinger, Sean C. Rush *The Learning Revolution*, Anker Publishing Company, 1997.
A. Ostebee, P. Zorn *Calculus*, Saunders College Publishing, 1997.
James Stewart, *Calculus*, Addison-Wesley 1996.
G. Thomas, R. Finney *Calculus, Early Transcendentals*, Brooks/Cole, 1991.

An Anti-Essentialist View about ICT in Mathematics Education: what Differences can it Make to Mathematics Teacher Education

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This paper reports both, the talk I gave at ICTME 2000 – Lebanese American University - Beirut, and my reflections after interacting and exchanging views with the ones who attended it. In the talk I outlined my PhD project and discussed some of the preliminary analysis of part of the data collected with respect to the case study one - Simon.

A PhD project:

Aims:

The aims of my PhD project are:

(1) To look at what is actually being said by secondary mathematics teachers about Cabri-Géomètre and Excel, i.e. to look at what meanings are being produced by teachers for Cabri and Excel; and

(2) To investigate to what extent those meanings are linked to the teachers' use of Cabri and Excel as well as to their teaching approaches.

Frame:

Two theoretical avenues have framed this project:

(1) Treating software packages (Cabri and Excel) as texts [1] and not as an essence, i.e. with no essential properties; treating users of software packages as readers, in the case of this project, secondary mathematics teachers as readers; and

(2) Meanings being produced [2] by secondary mathematics teachers for such texts: Cabri and Excel.

Focus:

Having as the focus of the project meanings being produced by teachers for Cabri and Excel implies to elicit the objects that are constituted by teachers for such software packages. The relevance of it, and one of my assumptions, is that the software package, which reaches the classroom environment, is not the software that once has been designed but rather software. A software package is the one that the teacher has constituted. For instance, it is not the Excel that is presented in a classroom but an Excel: the Excel of the teacher.

Design:

The samples of the project are secondary mathematics teachers from state schools in and around Bristol (England) for each of the software packages. From the data collected so far, it seems that 2 teachers for each software package are rich enough to carry out the study. This is something still under evaluation.

The main study was structured as follows:

Interview 1 (audio-taped): general questions about their first use of computer; their view about the use of computer in education and the view and use of Cabri and Excel in mathematical education.

Outcomes: profile of the teacher and overlaps to later collections.

Interview 2 (video-taped): in front of the computer with the teacher showing her/his Cabri/Excel.

Outcomes: the Cabri/Excel of the teacher and a worksheet that the teacher has developed.

Interview 3 (video-taped): in front of the computer with the teacher tackling a given problem.

Outcome: the teacher's use of her/his Cabri/Excel to solve the problem.

Worksheets (audio-taped): the teacher showing and talking about the worksheets s/he has designed and the ones available at school.

Outcomes: how the teacher works with the worksheets; how the worksheets are divided in levels; and how the teacher plans the mathematics lessons in the computer room.

Classroom observation (audio-taped): what the teacher says about Cabri/Excel; how s/he presents her/his Cabri/Excel to the pupils; and how the teacher conducts the lesson.

Outcome: the patterns of interactions of the teacher with some of her/his pupils with respect to the Cabri/Excel of the teacher.

first analysis: case study one

This preliminary analysis of the case study one - Simon - only involves:

(1) Interview 1 - which provides the profile of Simon;

(2) Interview 2 - which provides the Excel of Simon; and

(3) Five of his lessons observed - which provide the presentation of the Excel of Simon to the class and his interaction with some of the pupils.

<u>Profile of Simon:</u>

Simon is 28 years old and has been teaching secondary mathematics for five years at the same school. Since Simon started teaching at this school he has been using Excel in his teaching. Simon first used a computer when he was eight. He worked more with computers at home and with friends, than at school. Simon and his friends first started using computers for playing games and later on for programming in Basic. Simon graduated in Mathematics and Computing at Bath University. At that time, according to him, he worked with computers on a more theoretical level than on a practical one. He used programming in Assembler and C languages.

Simon plays an active role in his Mathematics Department. He always pushed his colleagues hard for using computers in their teaching. The Mathematics Department is responsible for introducing Excel spreadsheet to the school pupils and this is done in the Year 9 (12-13 years old). In the departmental meetings, Simon and his colleagues work on Excel worksheets and discuss planned mathematics lessons in the computer room.

Simon sees the use of computers in education valuable "in terms of doing things more quickly". (int. 1, 172)

Simon finds the use of Excel in his teaching useful and important because "it can open up higher level of mathematics for pupils". (int. 1, 372)

The excel of Simon:

Something about his text

"Excel is a big table that can do maths for you … there are lots of boxes in this table and I would normally call the boxes, cells." (int. 2, 004)

"I usually describe a cell as box that you can put three things in it: you can put a label that is text; you can put a number; or you can put a formula. And we now are probably talking about ... you just type some labels as a letter; if you want a number you type a number and if you want a formula you type equals ... the clever thing is using cell references, so each box has an unique name ... so the one I am on at the moment is B4..." (int. 2, 547)

"All sorts of stuff you can do in formatting the cell, in particular … we were looking at the number earlier … you might want something in a number of two decimal places or as we have done today, in pounds … you might want a date in a box or a fraction or time. Some number to two decimal places, you type in a cell like that. You want that particular box to record two decimal places, you click into the box, telling that you are going to use it: Format - Cells - Number - Two decimal places … which is useful for some of the maths we do. Again, from cells you can trace some forms, different fonts. We talked about borders earlier … you could put things on the left of the cell, the right, on top or bottom. All sorts of different format you might want to do…" (int. 2, 241)

Its use

"For functions, transformation of functions, differentiation." (Short questionnaire)

"... We can even get it (Excel) to solve some questions that it might find difficult, particularly if you are looking at trial and error." (int. 2, 114)

"... one of the most important uses of Excel spreadsheets is you change a number and if you set the rules correctly, all the answers after it change..." (int. 2, 004)

Reasons for using it

"... Save yourself a lot of time and effort..." (int. 2, 004)

"... It (Excel) can open up a higher level of mathematics for them (pupils) because they're getting away from some sort of slow drudging work they were doing before, spreadsheets can do the similar thing ... but it's also not just they've learned something to my main aim to the maths but also a few points about how to do things and well on Excel, graphs being one of them ... about formatting graphs..." (int. 1, 398)

His excel and interactions:

Simon gave two worksheets to the class: Rich Aunt (appendix 1) and Soup Cans (appendix 2).

The first worksheet lasted 2 lessons. Simon emphasised the use of insert columns, numbers as formulae, fill down and drawing a graph (xy scatter, scale and label) to the class. Any problem the pupils had, Simon stopped the lesson and drew the attention of the whole class to the objects of his Excel by stating the importance of them.

The second worksheet lasted 3 lessons. Simon emphasised the same objects of his Excel to the class: insert columns, insert formulae (volume), fill down and drawing a graph (xy scatter, scale and label). Again, any problem the pupils had, he stopped the lesson and called the attention to the whole class by showing things on the big screen. What interestingly happened during these lessons is that, although many of his pupils got wrong and 'stuck' when typing (entering) the volume formula (many of the pupils typed the formula with one bracket, three brackets and so on…) and therefore not getting the graph, Simon did not stop the lesson to call the whole class attention to that but rather said to these pupils, individually, how many brackets they should be putting in or taking out. I found this episode quite peculiar but when reading the transcriptions of interview 1 and 2 I understood how much 'algebra stuff' is not in the Excel of Simon, as you can notice below.

What is not his text:

"... (Format - Sheet - Hide) I don't really think of any at the moment. I can't remember what its use was, the need of a sheet to be hidden..." (int. 2, 333)

"... I am not quite sure much how useful are some of the functions (Insert - Function) that Excel could do ...hum... all this stuff (Insert - Name) has to do with ... I never really use that (name cell), to be honest ...hum... I think if you get into that what you can do is ... it is useful for algebra stuff, you could call a certain box X or something rather than B2 and then you use that ... I never really ... I've got papers about it somewhere."(int. 1, 241)

About properties of his excel

" ... One thing you can very easily do is underestimating the amount of knowledge needed to just basic start with spreadsheets ...hum... so, quite often the staff ... one of the problems that we have, particularly when we are trying to teach pupils IT, particularly the lower groups, their actual mathematical ability prevents them making much progress with spreadsheets ... because it's almost straight forward the spreadsheets ... talk about formulae you are talking about generalizing and it's quite a big jump for some of those pupils at that age to generalize in steps. Some of them it's a great way to make that step because, you know, they know about computers, they are computer literate, they can see..." (int. 1, 431)

"The clever thing is to use cell references ...hum... it's like a level of indirection, really...when you use the number in E4 and multiply the number in E5, which is in E4 (shows that)..."(int.2, 547)

" ... That's why spreadsheet is so powerful because you use the cell references and you can use them for three things: labels; numbers or formulae but the important thing it that each cell has its own address or reference that you can use." (int. 2, 553)

What I find quite interesting here is how much 'a non-algebraic language' Simon uses when defining the use of cell references in his text. It is quite interesting to define it as "a level of indirection". Moreover, it seems that for Simon "getting stuck" with formulae and generalisation is something linked to the pupils' knowledge about computers. The ones, who know about computers or are computer literate, can 'see', can make the step. (int. 1, 431).

Summing up and final remark:

If I call algebra and functions or transformation of functions as semantic fields, I would claim that it is quite important to be aware of what semantic field one is operating when producing meanings for a software package. Depending on what semantic field one is operating, a different text will come up. Hence, I argue that it is quite important to elucidate what text we are looking at. Thinking in terms of mathematics teacher education, being a teacher of teachers, it is crucial, in my opinion, to make explicit what semantic field one is operating in when teaching the use of a software package. More than that, to show and work on different semantic fields with teachers and show them different texts that one can constitute.

What I have just said above gives a flavour of the line of thinking I am taking to analyse the data. I am still working on it, by reading over and over again the transcriptions and linking all I am seeing in them with what I, theoretically speaking, have been thinking so far.

Acknowledgements:

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Celia Hoyles and Richard Noss for their sharp feedback following my talk.

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Abu Nadim who wonderfully introduced Beirut and Zahle to me as well as many great Lebanese people.

Endnotes:

[1] I am treating software package as text from an anti-essentialist view. Grint and Woolgar, both sociologists working in the field of Sociology of Technology, worked hard on defining an 'anti-essentialism move' about technology by treating technology as text, designers as writers and users as readers (Grint and Woolgar, 1997). To understand the anti-essentialism move it is necessary to talk about theories of technology and how certain approaches have been developed. I have written something about it elsewhere (Bibi Lins, in press.).

[2] The notion of meaning production and semantic fields are taken from an epistemological model developed by Lins (1992 and in press) for understanding what algebraic thinking is.

References:

Grint, K. and Woolgar, S.: 1997, The Machine at Work: Technology, Work and Organization. Polity Press.

Lins, B.: in press, The Importance of Premises: From An Essentialist to An Anti-Essentialist View of ICT in Mathematics Education, proceedings of the BSRLM Day Conference, Loughborough University, May 2000.

Lins, R.: 1992, A Framework for Understanding what Algebraic Thinking is, Unpublished PhD thesis, Nottingham, School of Education (Shell Centre).

Lins, R.: in press, The Production of Meaning for Algebra: A Perspective Based on a Theoretical Model of Semantic Fields, in Sutherland R., Rojano T., Bell A. and Lins R. (eds.), Algebraic Processes and Structure. Kluwer Academic Publishers.

Appendix 1:

Activity taken from Mathematics and Spreadsheets Workshop given by L. Healy and R. Sutherland (1992). "A Rich Aunt" is originally from SMILE 1425.

A Rich Aunt

Bob has a rich aunt, who is a mathematician. She wrote this letter to Bob:

Dove Cottage
Brainage
Hertfordshire
Telephone 241

Dear Bob

Now that I am getting on, (I'm 70 today), I want to give you some of my money. I shall give you a sum each year, starting now. You can choose which of the following schemes you would like me to use.

(a) £100 now, £90 next year, £80 the year after, and so on.

(b) £10 now, £20 next year, £30 the year after, and so on.

(c) £10 now, 1 and 1/2 times as much next year, 1 and 1/2 times as much again the year after that, and so on.

(d) £1 now, £2 next year, £4 the year after, £8 the year after that, and so on.

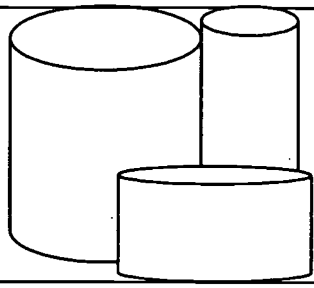
Of course, these schemes can only operate while I am alive. I look forward to hearing which scheme you choose, and why!

Best wishes Aunt Lucy.

Reply to this letter.

Appendix 2:

Soup Cans



Design some soup cans that contain 500 ml of soup. What is the minimum amount of tin needed?

Is it Just a Computer?

Christine Sabieh, Ph.D.

Notre Dame University, Lebanon

ABSTRACT:

Placing the computer within the teaching and learning environment is more than just introducing the computer into the educator's "math class". Computer assistance entails overcoming the fear of its use in the educator's "kingdom" and the educator's willingness to change. Being aware of how students learn, understanding the new role of the educator and the learner in relation to the computer, and realizing that restructuring is needed to integrate the technology into the environment are the grounds on which the educator can successfully use the computer as a tool or tutor to enhance the teaching and learning equation.

Placing the computer within the teaching and learning environment is more than just introducing the computer into the educator's "math class". Today's educational institutions have witnessed the introduction of the computer in the educator's "kingdom"—that is in the classroom, the laboratory setting or at home (Sabieh 1998; 1999a; 2000a; 2000b; 2000c) yet to a certain degree, the computer remains unused, misused or inefficiently used to help the teacher in his teaching and the students in their learning.

The rhetorical question is "why" has the computer remained so?

On the surface, it is rare that a teacher today will claim that he has not kept up with the changes in methodology to enhance his students' learning or his teaching skill. However, the fact remains, the educator, as many in his field—in Lebanon or abroad, resent the intrusion the computer has made into his "kingdom"(Sabieh 1998; 2000a; 2000b). It is such a negative or ignorant attitude about the computer and its role in education that tends to reinforce the dilemma of negative perception and abuse (Hawkridge, 1991; Sabieh 1998; 1999a; 1999b; 2000a; 2000c). As is evident at present that introducing the computer into the classroom will continue to be seen as a 'tool like all the other misused or inappropriately used tools' that already exist in the classroom setting. The educator and students alike will continue to use the computer as a typewriter, a figure organizer and presenter, a calculator, and a clean dust-less and muscle-easing tool. To position instructional technology in their teaching-learning medium remains abstract, senseless and mysterious at best.

The purpose of this paper is to show that a computer is more than just a computer. It is with this awareness that the educator takes on a partner to combat any given math's environment to ensure that his students learn effectively.

Based on the finding of Sabieh's 1998 study that the computer guaranteed a learning medium, that learning was possible with the aid of the computer, and that effective use of computer as a tool or tutor could bridge gap between student and learning outcome, Sabieh advocates computer assisted learning for any subject learning environment, specifically in this case the math environment. Accordingly, the computer is a tool of empowerment. It has the capability to organize, attract, and build confidence. It can supply different learning needs, provide immediate feedback, and evaluate learning. The computer is designed to integrate learning theories and conditions. It improves achievement, self-esteem, confidence and competence (Sabieh 1998; 1999b; 2000d).

Since 1999, Sabieh has been working with schoolteachers and groups of education students—future teachers of English, mathematics and science for the preschool, elementary and high school levels. She notes that some of the education students are already teachers in the schools. As the teachers and education students have learnt more and more about the role of computer in education, they have actively explored its use in "their" classrooms as a means to test the methodology, the software chosen and the planned activities. The teachers go back to their classrooms and explore the planned avenues with their students. The remaining education students do the trials with their peers. Both groups return to the learning sessions and provide the group as a whole with feedback to reinforce the work or to make modifications of what they have been working on. This, Sabieh notes, has proved to be a fruitful and rewarding experience to the learning sessions. It has promoted group work and discussion with the peers and within the whole classroom setting; the trainees have learnt to explore, experiment, support, and guide each other as a network would as they grow confident in their endeavor to promote computer assisted learning in their teaching. They have learnt by collaborating their views and experiences together (Crook, 1996).

Sabieh believes that the educator can successfully use the computer as a tool or tutor to enhance his teaching and his students' learning to ensure effective learning is taking place within the 'math class' by implementing three steps.

Sabieh believes that the first step to implementing computer assisted learning is to get the educator to overcome the fear of the computer. The educator, as any human being, fears the unknown, the unfamiliar, the uncontrollable. He fears the expectations of new ways, and the power change (Sabieh, 2000b). Moreover, he fears that the computer will change the teaching methodologies and curricula and that the educator cannot keep up with changes in computer world. Furthermore, the fear that the computer promotes unrealistic expectations in his teaching and that of his student's performance and that the computer will replace him are even bigger fears he experiences (Sabieh, 1998; 1999a; 2000b).

To overcome the fear of computer assisted learning, the educator must develop a clear understanding of its meaning. Computer assisted learning or computer based instruction means to use the computer to aid, help, instruct or guide work. Computer assisted learning does not mean to teach how to use of computer or to teach about computer technology. The computer can be used as a tool or a tutor or a tutee (Taylor, 1980; Anderson, 1991). As a tool, it is aids or facilitates the math work or learning of math in its environment. The tool is not to take the place of the maths educator or the tutor. Cunningham (1990) noted that in such a category classification, the computer acts as an instructional tool rather than a teaching tool. Its mastery as a tool to use is not needed alone. It is mastery of the computer with the computer program that is needed in the learning environment.

As a tutor, the computer is used to deliver instructional material in maths to enhance the students' learning. It aids the students in processing the information needed to master the math. Being more flexible than a textbook in its way of presenting the material, the computer as a tutor is considered to be an attractive medium to use with students who have difficulty in learning or understanding mathematics (Sabieh, 1998).

As a tutee, the computer assistance defines itself as a delivery system of knowledge that the students have input into the computer for later use. However, as the students input the information, they also learn their math simultaneously.

To further overcome the skepticism the educator feels about computer assistance, he must be made aware of the computer's impact in the learning environment. This can best be accomplished in four ways: Through education, through hands-on experience, through awareness of how the computer aids the learning environment, and by restructuring the education system (Ridgway, 1991; Sabieh, 1999a; 2000a). By learning about computer assistance, the educator becomes fearless and familiar with using the computer. The educator realizes that he is able to control it and work with it to make his teaching and his students' learning more effective. This message is acquired gradually through the hands-on experience he undertakes with education. Even if the educator knows how to use content-free software programs, such as word, excel, access, or lotus, this does not mean that he knows how to use them to enhance his students' learning. As the educator learns to do so, he is given immediate feedback on his work. He feels he is involved in his knowledge building. In this way, he is actively using the knowledge to enhance his ability and new understanding of the role the computer plays in education. Thus, the educator is becoming a confident end user. He is becoming more willing to use the computer in his "kingdom". Through the awareness of how the computer aids in learning, the educator learns to vary his methodologies, his lesson plans, and his choice of programs based on the needs of his students to enhance their learning. He is now aware of the need to restructure the education system to be able to implement the use of computer assistance to enhance his students' learning. He is consciously committed to bring about policy change, redefine objectives, set new goals, and create new curricula. He is aware that the computer has an impact as a delivery system, a motivator and a power tool in his "kingdom".

The educator can use the computer to improve quality of his instruction/ teaching to enhance the students' learning. He can present with the computer as tool. He can accomplish tasks by bringing the time, places, people, and events alive. He can connect with other educators & learners (Hennessy, O'Shea, Evertsz & Floyd, 1991; Sabieh, 2000b). The computer can be used to enhance the learning of different levels in one class (Sabieh, 1995; 1998; 1999b; 2000c). In short, the educator with the assistance of the computer can motivate, stimulate, and challenge the students in their learning environment.

The second step to implement computer assisted learning is to ensure that the educator understands how the students learn and how the roles of educator and students change in the new environment. Computer assisted learning can be used to teach mainstream students, students who are high or average or low achievers, and students with different needs, such as the learning disabled, the mentally handicapped, the behaviorally disturbed and the emotionally disturbed students (Sabieh 1999b; 2000c). Ideally, and in accordance with Sabieh's learning equation (1998), the students learn when they face learning activities, when they determine their needs, when they are driven and motivated, when they are actively involved, are reinforced and are given feedback on their work, and when they form habits and learning patterns in the math environment (Sabieh, 1998; 1999b). Add to the learning equation a new learning environment; one that is more learner-centered, communicative and content based. Globally, this has been the case for years; however, Lebanon has been undergoing changes in its curriculum for the past two years (Ministry of Education, 1997). The equation must be restructured to accommodate any change. Computer assistance, Sabieh stresses, is a solution to meet the shortcoming in the system to ensure that the students' learning will not be affected.

Given the 'new' educational setting, the computer reinforces the new active roles the educator and the students have to adopt in the communicative learning-environment. Both the students and the educator take on their new role and become empowered, and they take on a more active role in the classroom environment. The students become active learners, and the educator becomes free to explore avenues created in the communicative content-based classroom (Edwards, 1991; Fraser, Burkhardt, Coupland, Phillips, Pimm & Ridgway, 1991; Sabieh, 1999a; 2000a; 2000c; 2000d).

The students, with computer assistance, become active and responsible for their own learning. The students work at their own pace in the non-threatening environment with no cultural barriers. They are motivated and challenged to apply them selves to meet and accomplish their own learning needs. They are able to practice math concepts, and build and master skills to reinforce math structures. They receive immediate feedback and reinforcements on their math work to modify, correct errors and mistakes, and enhance their subject mastery. The students are able to imitate, explore, create, and develop the skills needed for functional math use. The use of the computer to assist the students in their learning promotes their self-concept, self-esteem and strengthens their confidence in the endeavor to learn mathematics. With computer assistance, the educator's new role becomes that of a stimulator and motivator, an organizer and mediator between the computer and the students. The educator becomes the facilitator of instruction—the resource consultant. He is the one who encourages and supports the students in their learning. It is the educator who decides on the purpose of computer use. Plus, he is the one to choose the learning environment, be it the classroom or the laboratory (Malone, 1993; Sabieh 1998). He is the one to choose the methodology, be it for remedial or mastery purposes, for the development or reinforcement of the underlying skills that underlie the communicative environment. He is the one who decides if the work is to be done for individualized learning and for cooperative group learning (Johnson & Johnson, 1986; 1994; Crook, 1996; Sabieh 1998). He also is the one who decides the type of activity to be undertaken; is the activity to be drill & practice or tutorial, problem solving, simulation or games, or content free software activities (Bramble, Mason & Berg, 1985; Geisert & Futrell, 1995; Sabieh, 1999a; 2000b).

In short, the educator in his new role is the one who controls how his students' learning is enhanced through computer assistance. The educator becomes aware that he is the agent of change in the educational system. He is the one who consciously takes on the task to enhance his students' learning through his teaching and through the use of his new partner—the computer.

The third step to implement computer assisted learning is for him to recognize himself as *the* agent to bring about the needed change in the educational system to improve the level of learning. This is because *he* is the one who is in close contact with the students (Sabieh, 2000d).

The educator must *consciously* be willing to put pressure on the university and school administrators and the Ministry of Education to meet the needs to implement the change to enhance the students' learning. (Sabieh, 2000b).

To do so, the educator must acknowledge the computer's importance and the useful role it has in the learning environment. In turn, the administrators and Ministry of Education must be ready to provide the educator with the new climate—the new set up, the support system and the training (Hawkridge, 1991; Sabieh 2000a; 2000b, 2000d). The learning equation is then reconstructed to integrate computers into it since computer assistance cannot just be introduced into an existing system with the expectation that the educator will modify the curriculum (Ridgway, 1991; Pearlman, 1993; Underwood & Underwood, 1995; Sabieh, 1998).

With the help of the educator and the administrators, the needs of the students, the educator and the educational setting are identified, clear objectives are rewritten and a curriculum is redesigned to meet the new needs to enhance the students' math acquisition.

With the implementation of the three steps, Sabieh believes, the ground will be fertile for the educator to successfully use the computer as a tool or tutor to enhance the teaching and learning equation to ensure effective learning is taking place within the 'math class'.

References:

Anderson, J. (1991). The computer as a tutor, tutee, tool in reading and language. In O. Boyd-Barrett & E. Scanlon (Eds.), *Computers and Learning* (pp. 39-50). Wokingham, England: Addison-Wesley Publishing.

Bramble, W. J., Mason, E. J. & Berg, P. (1985). *Computer in schools*. New York: McGraw-Hill.

Crook, C. (1996). *Computers and the collaborative experience of learning*. London: Routledge.

Cunningham, D. (1990). Past predictions, current perspectives, and future prospects in CALL. *Babe—Journal of the Australian Modern Language Teachers' Association, 25* (2), 3-26.

Edwards, D. (1991). Discourse and the development of understanding in the classroom. In O. Boyd-Barrett & E. Scanlon (Eds.), *Computers and Learning* (pp. 186-204). Wokingham, England: Addison-Wesley Publishing.

Fraser, R., Burkhardt, H., Coupland, J., Phillips, R., Pimm, D. & Ridgway, J. (1991). Learning activities and classroom roles with and without the microcomputer. In O. Boyd-Barrett & E. Scanlon (Eds.), *Computers and Learning* (pp. 205-228). Wokingham, England: Addison-Wesley Publishing.

Geisert, P. G. & Futrell, M. K. (1995). *Teachers, computers and curriculum: Microcomputers in the classroom.* 2nd ed. Needham Heights, Mass.: Allyn & Bacon.

Hawkridge, D. (1991). Creative gales and computers in third world schools. In O. Boyd-Barrett & E. Scanlon (Eds.), *Computers and Learning* (pp. 23-38). Wokingham, England: Addison-Wesley Publishing.

Hennessy, S., O'Shea, T., Evertsz, R. & Floyd, A. (1991). An intelligent tutoring system approach to teaching primary mathematics. In O. Boyd-Barrett & E. Scanlon (Eds.), *Computers and Learning* (pp. 292-309). Wokingham, England: Addison-Wesley Publishing.

Johnson, D. W. & Johnson, R. T. (1986). Computer assisted cooperative learning. *Educational Technology, 26* (1), 12-18.

Johnson, D. W. & Johnson, R. T. (1994). *Learning together and alone: Cooperative. competitive and individualistic learning* (4th ed.). Boston: Allyn and Bacon.

Malone, M. N. Jr. (1993). Classroom or lab: How to decide which is best. In T. Cannings & L. Finkel (Eds.), *The technology age classroom* (pp. 305-308). Wilsonville. Oregon: Franklin. Beedle and Associates, Incorp.

Ministry of Education curriculum Act of May 8th, 1997, Decree No. 10227. Beirut, Lebanon.

Pearlman, R. (1993). Restructuring with technology. In T. Cannings & L. Finkel (Eds.), *The technology age classroom.* (pp. 612-617). Wilsonville, Oregon: Franklin, Beedle and Associates, Incorp.

Ridgway, J. (1991). Of course ICAI is impossible.., worse, though, it might be seditious. In O. Boyd-Barrett & E. Scanlon (Eds.), *Computers and Learning* (pp. 123-146). Wokingham, England: Addison-Wesley Publishing.

Sabieh, C. (1995). Differentiated instruction: a key to handling individual differences, *Palma Journal,* pp. 131-147.

Sabieh, C. (1998). *The use of the computer as a tool and/or tutor to enhance language learning.* PhD Dissertation, Université du Saint-Esprit Kaslik., Lebanon. Manuscript submittef for publication.

Sabieh, C. (1999a, March) *Is computer assistance the solution to enhance learning?* Paper presented at the *Education Formation: Facts and Ambitions* Teacher Training Conference, Yarmouk University, Irbid, Jordan.

Sabieh, C. (1999b, July). *The psychology of learning and the use of the computer: A link to a solution.* Paper presented at the National Communication Association's Summer conference on Communication and Technology, Virginia, USA.

Sabieh, C. (2000a, May-June). Computer assistance: The solution to enhance learning. In A. Al-Riyani (Ed.), *The non-Governmental Higher Education Conference: Vol. 3, Non-Governmental Higher Education and the Futuristic Horizons* (pp. 183-206). Sana'a, Yemen: Queen Arwa University Press.

Sabieh, C. (2000b, May) *Computers in education: Is fear the problem?* Paper presented at the *Practical Teaching ideas for a New Millennium* SPELT TIE-IN, Damascus, Syria.

Sabieh, C. (2000c) The psychology of learning and the use of the computer: A practical solution. In H. E. Klein (Ed.), *Creative International Teaching: Case Method and Other Techniques* (pp. 231-245). Boston:WACRA.

Sabieh, C. (2000d, May) *To integrate the use of computer into Lebanon's new English curriculum as an aid to enhance learning.* Paper presented at the *New Lebanese Curriculum for Language: Arabic, English and French* conference, Notre Dame University, Zouk Mosbeh, Lebanon. (Submitted for publication)

Taylor, R. (ed.). (1980). *The computer in school: Tutor, tool. & tutee.* New York: Teacher College Press.

Underwood, J. D. M. & Underwood, G. (1995). *Computers and learning.* Oxford: Blackwell Publishers.

Mathematical Technology: In the Hand or on the Desktop?

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ABSTRACT:

Although mathematical technology is commonplace now, there is still controversy over, for example, the merits of various technologies both hand-held or desktop, the best ways to use such powerful technology to teach students, and the impact it has on educational practice, for example, with assessment. In this paper we discuss some of these points, in the light of a decade of continuing experience of use of technology with students, as part of the ongoing T-TIME project.

Introduction:

The use of mathematical technology in education is now established in many countries. In UK universities, students use technology to support both "doing" and learning mathematics. However there is still considerable debate and controversy over the precise role which technology should play in mathematics education. This revolves around a variety of issues, for example:

- The relative merits of different technologies, either software based on desktop machines, or hand-held machines.
- The extent to which students should be allowed or encouraged to use mathematical technology while learning, and at what stage.
- The way in which technology is changing traditional assessment practices.

In this paper we report some of our experiences concerning these matters, with particular emphasis on the first point, although inevitably this will cascade into the other two points. We base our discussion upon a decade of experience of developing technology-based approaches to mathematics at our university, under the auspices of the T-TIME project <http://www.shu.ac.uk/maths/>. This includes experience of running a modern Mathematics degree programme designed to implement technology appropriately, as well as teaching mathematics to various other students, in particular those studying engineering.

Using some mathematical examples we illustrate particular points, concerning tool selection and how students interact with those tools, and the need for precision and clarity in statements about desired learning outcomes. Critically, mathematical technology possesses strong visualisation features and the possibility of providing a motivational context. Furthermore machines such as the TI-89 provide significant inexpensive, portable, and therefore personal and constantly available processing power. Our experience indicates that once a decision is made to integrate technology into a course, significant opportunities arise to enhance a student's mathematical experiences, and both curriculum and assessment are significantly affected.

This paper is part of a continuing series looking at the effect of technology upon mathematical educational practice, including teaching, learning and assessment issues, and indeed what constitutes "doing mathematics" and "solving" a problem now that technology is here. (e.g. Gretton et al 1997(1), Gretton et al 1995)

Initial discussion:

The UK Quality Assurance Agency, which reviews the quality of all higher education provision, asks questions concerning the clarity of the learning outcomes specified for each course of study. What the students learn depends on the approach you take with them, what you ask them to do, and how you assess them. Here we use the topic of maxima and minima to make some points about this. If this topic is simply listed as such, it could be interpreted in a variety of ways. When students arrive on our courses, if you ask them to find a maximum, in general they immediately look for something to differentiate, and have met rules concerning second derivatives, which allow maxima and minima to be distinguished. They find problem formulation difficult, and often pass by graphical approaches, which we consider to be particularly obvious when there is technology available with powerful visualisation facilities.

In order to broaden experiences about a topic such as this, we explicitly advise them to "go for a ***SONG***" (Challis et al, 1997, Challis et al, 1999). This makes the point that they should address the richness of a concept, by combining ***S***ymbolic, ***O***ral, ***N***umeric and ***G***raphic approaches. These points are expanded below, but it is worth noting at this stage that the ***O***ral aspect is paramount. This includes problem formulation and discussion, as well as communicating the answer appropriately, and using mathematical language confidently, and without this aspect, how can a student be confident that they have a valid answer, or persuade others of this fact? This may be stating what has always been important with mathematics, but the point now is that easily available personal technology allows for instance visual approaches in a way that has not been available previously.

Examples:

In order to illustrate the points we are trying to make, we take a look at three topics related to maxima and minima: one well-worn problem, one more advanced topic, and one concerning motivational context and the generation of mathematical ideas from real data.

Example 1: The maxbox problem

This problem is well known (perhaps even a cliche), but ideal for demonstrating the range of approaches possible. Briefly a rectangular piece of A4-sized (29.7 mm by 21 mm) cardboard (a flatpiece of silver, if you can afford it, is more exciting!) has four equal squares removed from its four corners, then it is folded to form an open-topped box (Figure 1). The question is to find the size of square (value of x) which maximises the volume of the resulting box.

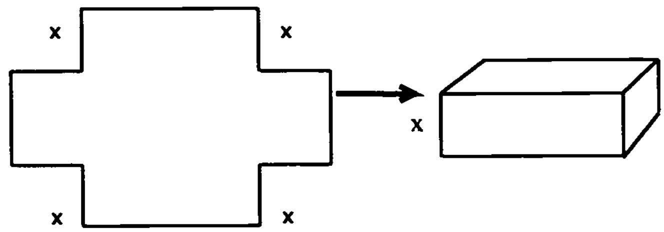

Figure 1

The formulation stage is important:

Volume $V(x) = x\,(29.7 - 2\,x)\,(21 - 2\,x)$

Once this expression is found and verified, the solution to the problem of finding the maximum can be carried out in a variety of ways, and can be rendered trivial by technology.

- *Symbolically by hand*

This symbolic processing approach is what many students regard as "mathematics", with other aspects being less important.

One solves $\quad V'(x) = 12\,x^2 - 202.8\,x + 623.7 = 0$

giving $\quad x = 4.04$ and $x = 12.86$.

The maximum is $x = 4.04$, classically identified by looking at $V''(x)$, yielding

$$V_{max} = 1128.50$$

Of course at the end of a process such as this, there must be some check carried out to make sure the answer is sensible. The process itself is not self-checking.

- *Graphically using a hand-held machine*

On any graphical machine (in this case a TI-83 Plus) we plot a graph of $V(x)$ against x and use visualisation features to "see" the maximum. This involves significant mathematical decision making, in choosing the appropriate scales and range, making use of key estimation skills, which are often ignored in symbolic approaches. This should probably be done in relation to the physical problem, thus developing skills in interpreting the context, and in applying the conclusions to that context. Most such machines have a TRACE feature, which can be used in conjunction with a ZOOM feature They also usually have built-in tools for seeking a maximum. After some thought, the range is set, the graph is drawn and the maximum is found as shown in Figure 2:

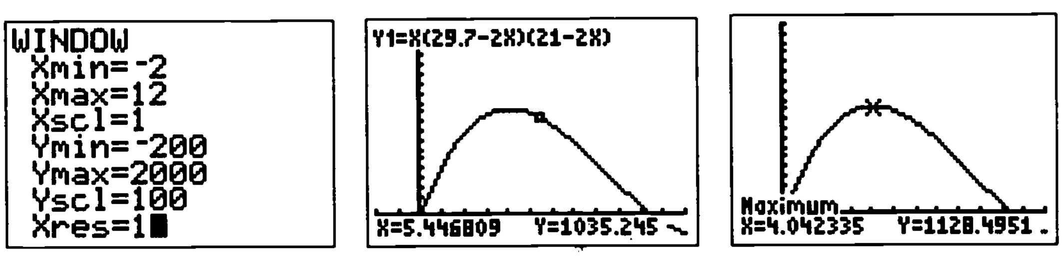

Figure 2

- *Using a Symbolic manipulator*

The algebraic approach can of course now also be supported through a *S*ymbol manipulation package such as Derive. Until recently such an approach required the use of a PC or equivalent, but the arrival of for example the TI-92 and TI-89 means that such technology is now portable and thus personal. The software can be used either to solve parts of the algebra, or to go direct to the solution (Figure 3)

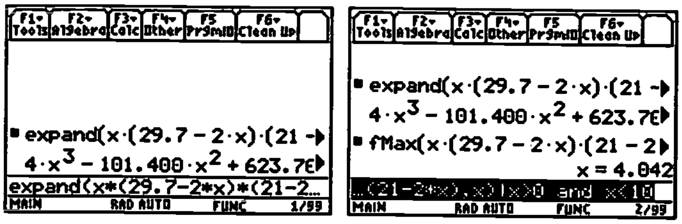

Figure 3

Note that again with this kind of approach one can build in estimation skills, in having to state the range over which the maximum is to be found, using the I symbol (Figure 3) to append "subject to..." conditions. Of course such packages or machines can also take the *G*raphical or *N*umerical approach.

- *Numerically using a table (Figure 4) or a spreadsheet (Figure 5)*

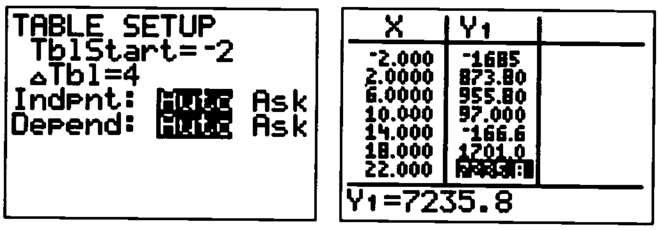

Figure 4

Note the table here can be adjusted to "zoom" into the solution area to whatever accuracy is required.

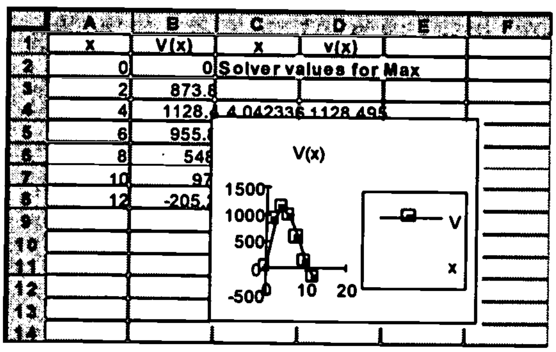

Figure 5

In this latter case, the maximum may be found as accurately as needed using the built-in Solver tool, which is a valuable tool and worthy of investigation for a wide range of problems for instance involving non-linear least squares model fitting.

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Example 2: Gibbs' phenomenon

This more advanced topic arises typically in the second year of a degree course. Traditional approaches are algebraic, with proof of the main result resting on subtle limiting arguments, and involving complicated, complex integrals. While mathematically rigorous, the complexity of this algebra can obscure what is happening conceptually, whereas a graphical approach can reveal the essence of the concept. An important distinction here has to be made between proving and convincing. The graphical approach described below does not constitute a proof, but gives meaning to the proof, a meaning which can be obscured by its complexity. This distinction is equally as important with an advanced topic as with more elementary work.

The square wave of period 2□ defined by

$$f(x)=\begin{cases}-1 & -\pi<x\leq 0\\ \ \ 1 & 0<x\leq\pi\end{cases}$$

may be represented by a Fourier series

$$f(t)=\sum_{n=1,odd}^{\infty}\frac{4}{n\pi}\sin(nt)$$

Gibbs' phenomenon arises in addressing the matter of convergence of the series when trying to represent a discontinuous function by an infinite series of continuous functions. Briefly each partial sum up to $n = N$ of the Fourier series will "overshoot" either side of the discontinuity, to a maximum value. In the limit as $N\rightarrow\infty$ the overshoot moves ever closer to the discontinuity. However it does not disappear but the maximum peak to peak value at the overshoot approaches in the limit a value approximately 18% of the size of the discontinuity. This attempt to describe the phenomenon in words is perhaps made clearer by the TI-89 screen shots in Figure 6.

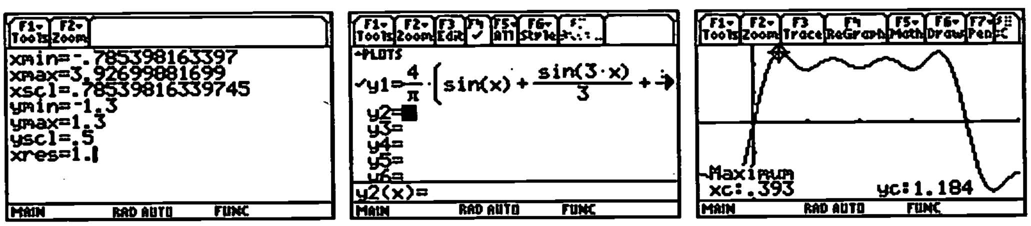

Figure 6

The phenomenon becomes visually obvious. The maximum nearest to the discontinuity can be found and evaluated by *G*raphical or *N*umerical means, and the effect can be observed for as many partial sums as necessary: Figure 6 is produced with S_7, for N = 7, and the settling down of the overshoot near 18% is already apparent. The idea is that conceptual development will be enhanced, and a later rigorous proof will be more transparent.

Example 3: Motivation from real data

The existence of inexpensive probes and data loggers such as the CBL™ and CBR™, used in conjunction with machines such as the TI-8*, gives opportunities to gather data simply and cheaply, and such data can provide a strong motivational context for the development of mathematical ideas. Figure 7 for instance shows some data for height against time gathered from an object thrown into the air, gathered using a CBL™ and motion sensor, and subsequently processed in a spreadsheet. Again one could ask what is the maximum height, as well as other questions. An important point is that the data is collected and owned by the students, so this is their problem, and will reinforce their concept by relating it to reality.

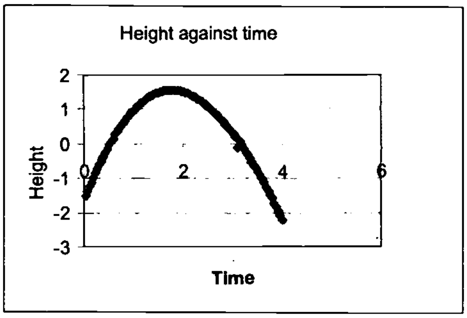

Figure 7

Further discussion:

The studies described above illustrate some interesting points. An activity which is ostensibly about finding a maximum can achieve broader aims, and demonstrate that we give our students the opportunity to develop skills in problem formulation, solving, checking for validity, and discussing and communicating appropriately in a range of ways. These skills do not develop without encouragement, or without our saying that we value them, and the only way we can really show we value them is to award some marks through course assessment.

On the specific issue of the merits of various types of technology, there are several points we wish to make. Cost is of course important, and hand-held machines are cheaper than a PC. They carry the additional advantage that they are personal, portable and therefore always available. The link with data logging equipment provides a significant bonus even for students of mathematics, with a significant recent advance in this respect through the TI-Interactive package <http://www.ti.com/calc>. In our experience mathematics degree students need a motivational context just as much as do those on more practical courses such as engineering.

There are other issues to deal with. The small screen has already been mentioned. Another issue is assessment. Portable technology can necessitate change in traditional examinations: either its use is allowed, in which case some traditional questions become trivial; or it is not, in which case there is an educational difficulty with the fact that students who are used to using such machines are disadvantaged! Our response to this has been to change the nature of our examinations, and make more significant use of coursework, although this kind of approach can lead to a heavy marking load and the possibility of plagiarism.

Some of our evaluation reveals that some students express the need to "learn the basics properly". Some academics in previous seminar discussions have expressed the view that having done this, students should go straight on to use the professional packages such as Mathematica or Maple. This provokes two comments: that hand-held machines, while relatively inexpensive, are not trivial or exclusively low level, as is apparent from Example 2; and that used correctly technology can enhance the learning of the basics. There is clearly some convincing still to be done though, both of our colleagues and some of our students.

To conclude this discussion we return to a previous point. The way a topic is approached should be determined by what students want to gain. This will also help to determine the role and type of technology to be used. Time spent on addressing all aspects of a problem, from formulation through checking to communication of the solution, or on studying a topic from several directions in order to enrich the concept, is time not spent on "traditional" activities. Perhaps the amount of content needs to be diminished - but is it a dilution of standards to reduce content in order to study less but more deeply? Indeed does technology give us the possibility of having the best of all worlds?

Conclusion:

The essence of the argument being presented here is that technology gives the possibility of a richer and more visual approach to mathematical concepts, as well as of creating an enhanced motivational context. Such a change is helped if the technology is personal to the student, and constantly available, and this seems to support the use of portable hand held machines. The distinction between these machines and desktop machines is becoming increasingly blurred in terms of the range of facilities available. Indeed we suggest that to some extent it ain't what you use, it's the way that you use it! (Gretton et al, 1997(2)) However the screen size of the hand held machines is the limitation, which can be off-putting with applications such as dynamic geometry, or when long algebraic expressions are to be handled (see Figure 6).

Of course economic factors always play a crucial role. There are benefits to be gained from students having experience over a wide range of professional software-based tools, but also having their own personal hand-held technology. Tool selection is an important skill for students now, and we have first year undergraduates carrying out critical comparative reviews of a variety of mathematical technology. Strong visualisation features and portable, inexpensive data gathering tools such as the CBR™ and CBL™ can provide a strong motivational context for developing mathematical concepts, and we find that a TI-89 for instance can become a fully integrated part of the way students function as mathematicians. There are of course also dangers, for instance of possible over-dependence, and of the undermining of some valuable skills and of traditional assessment practice. We might ask though what basic mathematics is useful now that technology is here? What is evident is that once technology is utilised and integrated, then it is necessary to review both curriculum and assessment to ensure that the desired outcomes of learning are addressed.

References

Challis N V and Gretton H W, *Assessment: does the punishment fit the crime?*, Proc. ICTCM12, San Francisco, Addison-Wesley-Longman, 1999 (to appear)

Challis N V and Gretton H W, *Technology, key skills and the engineering mathematics curriculum*, Proc. IMA conference on Mathematical Education of Engineers, (IMA), pp 145-150, 1997

Gretton H W and Challis N V, *I like coffee, I like tea*, Teaching Maths and its Applications, Vol. 14 No 4, 163-168, 1995

Gretton HW, Challis N V and Callender J T, *Teaching Engineering Field Theory using IT*, Int J Mech Eng Educ Vol 25 No 1 p1-12, 1997(1).

Gretton H W and Challis N V, *It ain't what you use, it's the way that you use it*, Spreadsheet User, Vol 4 No 2, pp13-14, Nov 1997(2) (ISSN 1362-1853)

Multi-Objective Optimization In Computer Aided Control System Design

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ABSTRACT

Control has a strong base in mathematics. The interaction goes in both directions, a wide range of mathematics has found its use in control, and control has occasionally stimulated the development of mathematics. Typical examples are optimal control, which revitalized the field of calculus of variations, stochastic control, and non-linear control. In this paper, a parametric expression of state feedback control, derived from the eigenstructure assignment approach of multi variable control system, was embedded in multi-objective optimization criteria to achieve some design aspects. *The multi-objective optimization, however, permits the separate design objectives to be simultaneously optimized allowing the designer to see how each objective affect the final solution. Also, it enables prioritization of the different goals so that more important performance goals are met at the expense of the less important ones. With the aid of optimization techniques, algorithms were developed and an interactive software package has been built up to be used in computer aided design of multivariable control system. The attendant computation burden of multi-objective optimization approach using parallel processing platform, attached to workstation hosting MATLAB is elucidated.*

Introduction

A major step towards the development of systematic design procedure for the eigenstructure assignment has been recently taken into a series of papers [1,2,3,4]. The parametric expression for the state feedback controller of the multivariable control system is an outcome of the procedure. The powerfulness of this parametric form comes from the ability to use their effective parameters, set of closed loop eigenvalues and the minimum number of free parameters [4], in satisfying design criteria. In other words, these parameters are available to the designer to be used to satisfy further requirements other than closed loop eigenvalues assignment, such as minimum gain control, satisfactory performance of the closed loop system or robust fixed gain ...etc.

The parametric expression and optimization techniques are cast together in a powerful algorithm, which is the base of an interactive software package developed for computer aided design of multivariable control system.

Parametric Control Formula Used In Computer Aided Control System Design

It is well known that, apart from the case of single input system, specification of the closed loop eigenvalues does not uniquely define a closed loop system. The source of non-uniqueness can be identified as that coming from the freedom offered by state feedback, beyond eigenvalues assignment, in selecting the associated eigenvectors and generalized eigenvectors from an admissible class.

A major step towards the development of systematic design procedure for the eigenstructure (eigenvalues and eigenvectors) assignment has been recently taken into a series of papers [1,2,3,4].

Consider the linear, time invariant, completely controllable and observable system

$$X(t) = A\,X(t) + B\,U(t) \qquad (2.1.a)$$

$$Y(t)=C\,X(t) \tag{2.1.b}$$

Where $X(t) \in R^n$ is the state vector,

$U(t) \in R^r$ is the control vector,

$Y(t) \in R^l$ is the output vector, and

A and B are real matrices of compatible dimensions.

In order to control that system, a linear state feedback control law

$$U(t)= K\,X(t) \tag{2.2}$$

Is applied to the open-loop system (2.1.a). The resulting closed-loop system takes the form:

$$X(t) = A_c X(t), \qquad A_c = A+ BK \tag{2.3}$$

The eigenstructure assignment problem is one of determining a real feedback gain matrix K such that the closed loop-system (2.3) is assigned an arbitrary self conjugate set of n-eigenvalues $\{S_i\}$, together with any associated eigenvectors and generalized eigenvectors. In essence, the parametric eignstructure assignment approach consists of parameterizing all the degrees of freedom available in the selection of the state feedback gain matrix by the n-eigenvalues $S_1, S_2, S_3, \ldots, S_n$ and n r-dimensional arbitrary parameter vectors $f_1, f_2, f_3, \ldots, f_n$.

The non-unique state feedback gain matrix K, which assigns the eigenvalues spectrum $\{S_i\}$ to the closed loop system (2.3), in assumption that the desired closed-loop eigenvalues $\{S_i\}$ are different from open-loop eigenvalues, is given by

$$K= FV^{-1} \tag{2.4}$$

Where F is an r x n parameter matrix defined as

$$F = [\, f_1 \;\; f_2 \;\ldots\; f_n] \tag{2.5}$$

And V= V(F) is an n x n modal matrix of the closed-loop system matrix A_c defined by

$$V = [v_1\, v_2 \;\ldots\; v_n] \tag{2.6}$$

Where v_i is the i-th closed loop eigenvector given by

$$v_i = (\, S_i \;\, I-A)^{-1}B\, f_i \tag{2.7}$$

Moreover the parameter vectors f_i, i= 1,2,..., n are arbitrary chosen under the condition :

(i) $|V| \neq 0$

(ii) $f_i \in R^r$ for a real eigenvalue S_i, whereas $f_j = f_i \in R^r$ for a complex conjugate pair of eigenvalues S_i, $S_j = Si^*$

as result the gain matrix K is expressed as

$$\mathbf{K=[f_1\; f_2 \ldots f_n][\, (S_1\, I\text{-}A)^{-1}Bf_1 \;\; (S_2\, I\text{-}A)^{-1}Bf_2 \ldots (S_n\, I\text{-}A)^{-1}Bf_n\,]^{-1}} \tag{2.8}$$

$$\mathbf{K=\phi[f_1, f_2,\ldots, f_n\, ;\, S_1, S_2,\ldots, S_n]} \tag{2.9}$$

Which indicates that the state feedback gain matrix K takes the explicitly parameterized form. The freedom in choosing the parameter vectors $\{f_i\}$ reflects the freedom offered in choosing the parameter vectors $\{v_i\}$ beyond the eigenvalues assignment $\{S_i\}$.

Multi-objective Optimization Design Criteria

The powerfulness of the parametric state feedback expression (2.8), comes from the ability to use their effective parameters in satisfying Multi-objective Optimization (MO) design aspects, such as, minimum gain output feedback control, satisfying performance of the closed loop system, and robust fixed gain control ... etc.

The traditional optimisation approach involves the aggregation of the different design approach into a single coast function. This obscures the impact of each design criterion on the optimisation process. MO, however, permits the separate design objectives to be simultaneously optimised. This allows the designer to see how each of objective function affects the final solution. Also, it enables the prioritisation of the different goals so that more important performance goals are met on the expense of less important ones. Mathematically, the process is described as [8].

$$\min_{\underline{x} \in \Omega} f(\underline{x}) \qquad (2.10)$$

Where $\underline{x}$ is the design parameter vector and Ω is the visible parameter space and f is the set of objective functions.

In our problem, embedding the parametric state feedback controller in a multi-objective design problem results in a list of performance criteria, the values of which depend on the free parameters sets $\{S_i\}$ and the parameter vectors $\{f_i\}$ of the closed loop control system respectively.

Some Design Criteria

i) Minimum Norm Output Feedback Control Design Under Specified Eigenvalues Areas

In most practical cases, the implementation of state feedback control law is impossible; since in general, only the output variables of the plant are available for control purposes. Assuming that all output variables are also state variables of the plant, structural constraint of the controller to determine output feedback control law, as was suggested in [6], is determined by minimization of

$$J_s = \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n} g_j k_{ij}^2 \qquad , g_j > 0 \qquad (3.1)$$

$$= \frac{1}{2}\operatorname{trace}(KGK^T) = J_s(\{S_i\},\{f_i\}). \qquad (3.2)$$

Where k_{ij} are the element of the controller K with $G = \operatorname{diag}\{g_j\}$. g_j are the positive weighting factors by which such controller columns that corresponds to non-measurable state variable can be forced to become very small during the minimization. Consequently they can be neglected and the control law results in.

$$U(t) = -k_1 y_1(t) - k_2(t) y_2(t) \;\ldots\ldots\; k_q y_q(t) \qquad (3.3)$$

Where $y_1, y_2, \ldots\ldots, y_q$ are the state variables that are known by measurement.

Minimizing J_s with w.r.t the parameter vectors f_i, the gradients of J_s w.r.t. f_i are given [6] by

$$\frac{\partial J_s}{\partial f_i} = [I_m - K(A - S_i I_n)^{-1} B)^T]KGw_i \qquad , i = 1,2,\ldots \;, n \qquad (3.4)$$

Where w_i as the i-th reciprocal eigenvector of the closed loop system,

$$w_i = (V^{-1})^T e_i \; ; \; e_i^T = (0 \;\; 0 \ldots 1 \ldots 0 \ldots 0) \qquad (3.5)$$

Minimizing J_s w.r.t. the eigenvalues S_i, the gradients of the eigenvalues is given by

$$\frac{\partial J_s}{\partial S_i} = [0_m - K(A - S_i I_n)^{-1} v_i)^T]KGw_i \qquad i = 1,2,\ldots \;\; ,n \qquad (3.6)$$

During the minimization process each eigenvalues S_i of the closed loop system has to remain within a specified region of the eigenvalue plane in order to meet given dynamical requirements like stability, good damping, and acceptable speed of response. Thus a constraint parameter minimization problem arises. It can be reduced to an unconstrained one by the transformation [5,6].

$$\hat{S}_i = \frac{1}{2} \ln \frac{S_i - \alpha_i}{\beta_i - S_i} \qquad i = 1,2,\ldots \;\; ,n \qquad (3.7)$$

Provided that the original eigenvalues S_i are subjected to

$$\alpha_i < S_i < \beta_I \qquad (3.8)$$

The eigenvalues gradients then are given by

$$\frac{\partial J_s}{\partial \hat{S}_i} = \frac{\partial J_s}{\partial S_i} \cdot \frac{\partial S_i}{\partial \hat{S}_i} \qquad (3.9)$$

ii) The criterion for the satisfactory performance usually is taken to be

$$J_p = \frac{1}{2} \int_0^\infty (X^T S X + U^T R U)\, dt \qquad (3.10)$$

Where $S \geq 0$ and $R > 0$

iii) The criterion for the robust gain control may be achieved by

$$J_i = \frac{1}{2} \sum_{l=1}^{n-1} \sum_{i=1}^{m} \sum_{j=1}^{n} (k_{ijl} - k_{ij0})^2 \qquad (3.11)$$

In the sense that the N-controllers for each of N-operating point must be equal.

Thus the result is a list of performance criteria the values of which depends on the values of the parameter sets $\{S_i\}$and $\{f_i\}$ respectively, that is

$J_i = J_i(S_1, S_2, \ldots, S_n; f_1, f_2, \ldots, f_n)$

Design Example

To illustrate the given procedure, the design of an output feedback control for a three tanks system is considered [6]. Its linear state equation was found:

$$X(t) = \begin{bmatrix} -0.332 & 0.332 & 0 \\ 0.332 & -0.664 & 0.332 \\ 0 & 0.332 & -0.524 \end{bmatrix} X(t) + \begin{bmatrix} 0.764 & 0 \\ 0 & 0 \\ 0 & 0.764 \end{bmatrix} U(t)$$

The open loop eigenvalues are –0.047, -0.437 and –1.36. They are to be shifted into the regions

$-0.4 < S_1 < -0.2$

$-1 < S_2 < -0.4$

$-2 < S_3 < -1$

By considering the first state variable X_1 alone. The weighting factors, in the performance index (3.1), are chosen as follow:

$g_1 = 1$ and $g_2 = g_3 = 10^6$

Starting from $S_{1o} = -0.3$, $S_{2o} = -0.7$, $S_{3o} = -1.5$, the numerical minimization yields the gain K^*

$$K^* = \begin{bmatrix} 0.742 & 3.6x10^{-5} & 2.9x10^{-5} \\ 0.202 & 1.2x10^{-5} & 1.1x10^{-5} \end{bmatrix} \cong \begin{bmatrix} 0.742 \\ 0.202 \end{bmatrix}$$

Hence, the following output feedback control law

$$U(t) = -\begin{bmatrix} 0.742 \\ 0.202 \end{bmatrix} X_1(t) = K_1 y_1(t)$$

Which shifts the closed loop eigenvalues to $S_1 = -0.202$, $S_2 = -0.636$, $S_3 = -1.23$.

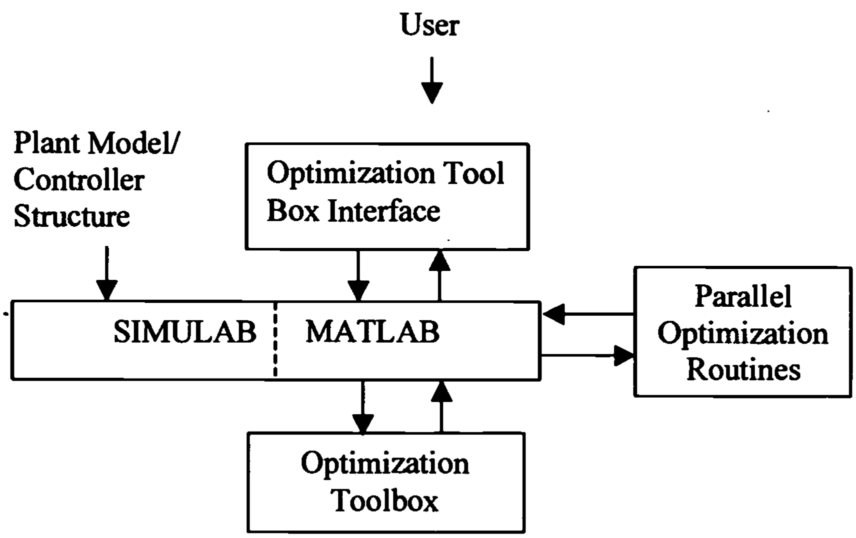
In addition, the controller is of small norm $\|K^*\| = 0.77$.

Computer Aided Control System Design And Optimization

To aid the designer, a user interface shell is developed [8], to integrate with MATLAB/SIMULAB which permits the automatic use of SIMULAB models and the easy expression of most common system design goals.

To make best use of this optimization approach, specially for non linear systems, it is necessary to be able to easily integrate the optimization software with a simulation package since the system may be simulated many times during each optimization process. The combination of SIMULAB and MATLAB gives the designer the ability to model and analyze non-linear systems in one environment.

The user interface runs on a UNIX workstation using X-windows (as does MATLAB) and is directly linked to MATLAB via MEX-file utilities of MATLAB. The interface fits into an optimization environment as shown in figure (1).



Here it can be seen that the interface communicates only with MATLAB/SIMULAB and calls MATLAB commands to perform all its functions. This architecture means that the implementation of the optimization commands is encapsulated and thus can be written as MATLAB M-file, C or FORTRAN MEX-files or as parallel routines on a parallel processing platform.

User Interface for Multi-objective Optimization (MO)

The interface is designed to allow the user to perform the following tasks:

i) Formulate the MO problems, where the control system (plant and controller) may be designed in SIMULAB block diagram.

ii) Specify the system design criteria in the time and frequency domain.

iv) View the results of the optimization in the same domain as the original criteria numerical or graphical.

v) Provide context-sensitive help to the user when required.

These tasks are implemented in a tool that interfaces closely with SIMULAB via the MEX-files facility provided in MATLAB and SIMULAB. The interface code is run as separate process to SIMULAB, with communication between the two being achieved via Berkeley UNIX-style Sockets.

The SIMULAB ends of the communication channel is handled by a MEX-file (see figure (2)). The interface process sends command and data to the communication MEX-file, which calls the appropriate MATLAB/SIMULAB functions. If results are expected the interface pr0cess will request them to be read back through the pipe. The use of MEX-file to handle communication offers advantages over the more usual method of interfacing with MATLAB/ SIMULAB. The advantages are that a proper communication protocol can be established between the two processes such that the synchronization can be maintained. The interface is implemented using C-language and X-windows Toolkits on a SUN-workstation.

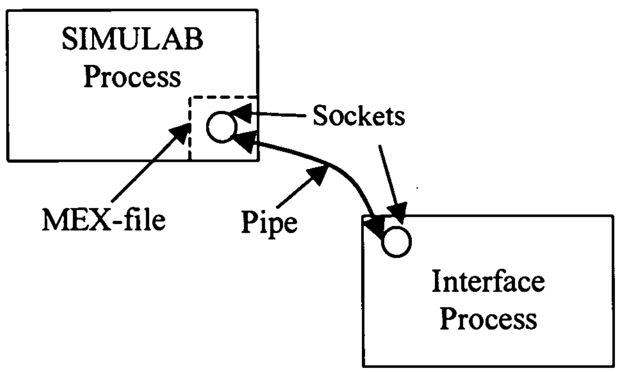


Figure (2): Communication between SIMULAB and Interface

Multi-objective Optimization Using Parallel Processing Platform Attached to Workstation Hosting MATLAB

Since the computation routines are called via MATLAB/SIMULAB, it is possible using the MEX-file interface to implement the various optimization routines on the parallel processing platform. One implementation strategy involves mapping the individual objective functions into separate processors. These objectives compromise the major part of the computation load of the optimization process since each one must be evaluated at every iteration cycle of the algorithm. A method of identifying and further paralyzing the more complex objective functions, at present is being investigated. The parallel processing of the optimization algorithm is achieved using MEX-files. When the parameters and functions for the optimization are defined, say, using the user interface a gateway routines, which is a MEX-file construct a message which is passed to the root parallel processing node. This then forms out each objective to the available processors. The gateway program passes the new estimate of the solution to the former node, which passes this one to the individual processor and collect the results for return to the optimization program (see Fig. (3)).

The hardware currently used in the environment compromises the SUN-workstation couples to the transputer-based platform consisting of n-parallel processors. The arrangement is shown in figure (4).

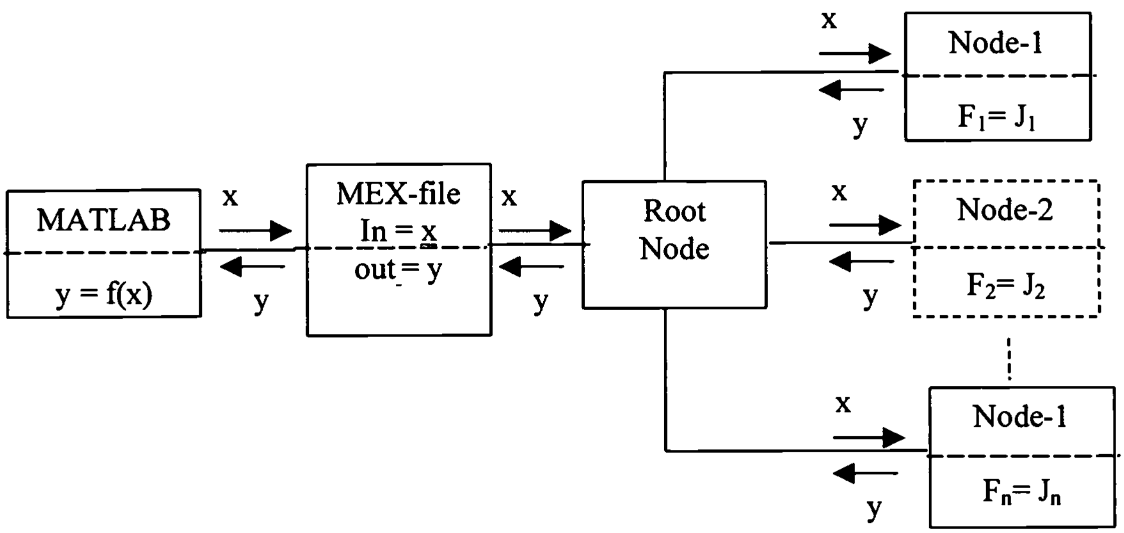


Figure (3) : Mapping objective Functions into Parallel Processing Node

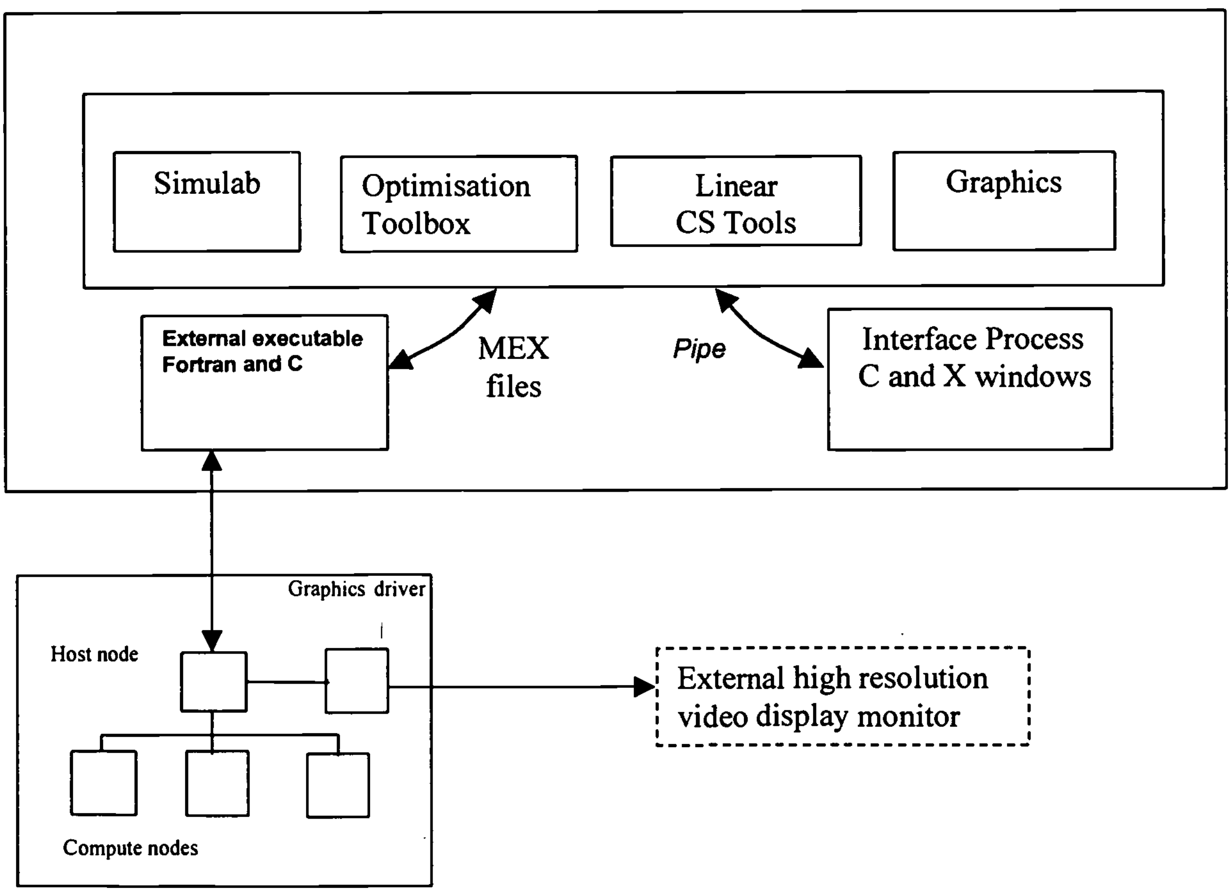


Figure (4) : MATLAB/transputer platform

Conclusions

1. A parametric expression for the state feedback controller was derived, by direct manipulation of the closed loop characteristic equation in such a way that inherit non-uniqueness is wholly displayed and conveniently organized. Such expression can be easily used in CACSD.

2. The expression is embedded in multi-objective design criteria, instead of aggregating the different design objectives into a single cost function.

3. The multi-objective optimization, however, permits the separate design objective to be simultaneously optimized allowing the designer to see how each objective affects the final solution.

4. To alleviate the computation burden, of the multi-objective optimization, a parallel processing platform attached to the workstation hosting MATLAB, is suggested.

References

[1] Fahmy M. M. and O'Reilly J., "*Eigenstructure Assignment in Linear Multivariable System*", IEEE transaction Automatic Control, Vol. AC-28, 1983, pp. 990-994.

[2] Fahmy M. M. and Tantawy H. S., "*Eigenstructure Via Linear State Feed Back Control*", Int. J. Control, Vol. 40, 1984, pp. 160-178.

[3] Roppeneker G., "*polvergable Durch Zusstandsruckfuhrung*", Regelungstechnik, Vol. 29, 1981, pp. 228-233.

[4] Fahmy M. M. and O'Reilly J., "*The Minimum Number of Degrees of Freedom in State Feedback Control*", Int. J. Control, Vol. 41, No. 3, 1985, pp. 749-768.

[5] Tantawy H. S. and O'Reilly J., "*Possiblities for Linear Feedback Control of Power system Using parametric Eigenstructure Assignment*", Fourth IMA Int. Conf. On Control Theory, Cambridge, 1984.

[6] Roppeneker G., "*Minimum Norm output Feedback design Under Specified Eigenvalues Areas*", Systems and Control Letter 3, pp. 101-103, North Holland, 1983.

[7] Kreisselmeir G.; Steihauser R., "*Systematic Control Design by Optimization a Vector Performance Index*", IFAC Symposium on Computer Aided Design of Control Systems, Zurich, Switzerland, 1979, pp. 113-117.

[8] Derek A. and Linkens (ED), "*CAD for Control Systems*", Marcel Deker Inc., 1993.

[9] Paul M. Frank (ED), "Advances in Control", HighLights of ECC 1999, 5th: 1999, KarLsruhe - Germany.

The Effect of Using the Geometer's Sketchpad (GSP) on Jordanian Students' Understanding of Geometrical Concepts.

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ABSTRACT:

Technology has become a part of most of our activities in the everyday life. It entered to the educational field as well as the other fields. The use of technology in schools is growing in both technological equipments like computers and the structure for them, besides the training programs for the teachers and other users. The new technological tools, such as computers and their software, provide people with more opportunities to teach in new ways. This environment of using technology is growing in the general reform in mathematics education.

The purpose of this study was to investigate the effect of using the Geometer's Sketchpad (GSP) on students' understanding of some of the geometrical concepts. The sample consisted of 52 students from the Model School, Yarmouk University, Jordan. The students in the experimental group used the GSP software once a week and the book, while the students in the control group used only the book. Both groups took the same pretest and posttest, which was designed by the researcher.

The results of the study indicated that there was a significant difference between the means of the students scores on the posttest with favor to the experimental group. The results also indicated that there were more gain in the scores from the pretest to the posttest in the case of the experimental group. The researcher suggested more use of the GSP software and more investigations in the area of using computers in education.

Technology is one of the major aspects in the educational process in all levels. It is "not only a product of a given culture; it also shapes the culture that created it" (Mehligner, 1998, p.8). The new technologies such as computers might affect the schooling system- if they are used in the right way- because technology provides learners (students) the power of controlling what they are learning. While teachers and administrators had the power in the past to determine what would be taught and what would not be taught.

Computers' use is in an increasing rate all over the world. The ratio of the number of students to the number of computers has been changed from 125:1 to 10:1 in the twelve-year span ending in 1996 in the United States (Quarterly Educational Data, 1996). "Growth is also an evident in the computer-related products ((Video-Disk Players, CD-ROM Players, Local Area Networks) and computer-independent forms of technology (Cable TV access was added in the nearly 20% of schools in three years- 76% of the US school districts have cable)" (Grabe and Grabe, 1998, p.10). Computers might be used to teach, to facilitate studying several topics, to help students to learn how to use technology, and to increase the effectiveness of performing academic tasks (Becker, 1991).

This study focuses on using one of the computer's software, the Geometer's Sketchpad (GSP) (Jackiw, 1991), on students' understanding of some of the geometrical concepts in mathematics. It gets its importance from the results of the 1994 study commissioned by the Software Publishers Association. Some of the conclusions of that study were the following:

Educational technology has a significant positive impact on achievement in all subject areas, across all levels of school, and in regular classrooms as well as those for special-needs students.

Educational technology has positive effects on students' attitudes.
Technology makes instruction more student-centered, encourages cooperative learning, and simulates increased teacher-student interaction (Mehligner, 1998, p.12).

The GSP is an interactive and dynamic computer program that can be used to help students learn and understand geometrical concepts and principles. "The GSP lets the user explore simple, as well as highly complex, theorems and relations in geometry" (Giamatti, 1995, p.456). It also "has the ability to record students' constructions as scripts. The most useful aspect of scripting ones' constructions is that students can test weather their constructions work in general or weather they have discovered a special case" (p.450). In addition, the GSP software provides the process of learning and teaching mathematics by a remarkable help because "the power of the GSP combined with the power of proof gives a complete illustration of the theorem involved and the aspects of "doing" mathematics" (p.458).

Students have many reasons for making a sketch with the GSP. "Their purpose may be to explore the behavior of a particular geometric figure, such as a rhombus, or to model a physical situation, such as a ladder leaning against a wall. They may want to make a beautiful pattern inspired by Navajo rug designs, or their goal may be an animation-perhaps a Ferris wheel or a merry-go-round" (Finzer and Bennett, 1995, p.128). The most important thing about the GSP Software is that GSP is an active dynamic program with a useful feature by using the mouse interface for graphics and high speed.

This study tried to answer the following questions:

1. Are there any differences between the means of the pretest and the posttest for the experimental group?

2. Are there any differences between the means of the pretest and the posttest for the control group?

3. Are there any differences between the means of the pretest for the experimental group and the control group?

4. Are there any differences between the means of the posttest for the experimental group and the control group?

Previous Studies:

Because the Geometer's Sketchpad was discovered in the recent years, only few research studies were done in its area. In the same time, most of the effort that was done, so far, focused on providing the teachers and students by some good examples of how using the GSP software in mathematics. Here is a summary of the some of these studies.

Dixon (1996) conducted a study and concluded that students who used the GSP (dynamic instructional environment) had higher significant achievement scores on a test containing the concepts of reflection and rotation. Groman (1996) studied using the GSP in a Geometry Course for Secondary Education Mathematics Majors and offered three examples of how sketchpad is used. The findings of the study showed that students wanted to get their own copies of the GSP software. The use of the GSP showed more positive reaction from both the students and the instructors in testing conjectures and constructions.

Youssef (1997) conducted a study to investigate the effect of using the GSP on the high school students' attitudes towards geometry. One of the results of that study indicated that the scores of the pretest and posttest of the students in the experimental group were significantly different. Another result indicated that there was a significant difference between the control and experimental groups in the gain of the scores from the pretest to the posttest.

Lester (1996) conducted a study to investigate the effects of the GSP software on achievement of geometric knowledge of high school geometry students. The results indicated that the mean of posttest scores for the dependent variable (geometric conjectures) of the experimental group was significantly higher than that of the control group. According to the same study, the GSP provides intelligent capabilities for improving learning and teaching. In addition, Lester's study raised the issue of preparing qualified teachers in using the new technologies and software in an effective way.

In general, the results of the studies and the discussions about the use of the GSP in teaching and learning mathematics indicated that it is a useful and attractive program that can create a healthy atmosphere in the educational process. Because using this program will provide students by a good chance of simulation, which is very close to the real life situations. In addition, Mehligner (1998) stated that in order to get the maximum benefit from technology, "schools should expect more integration, interaction, and intelligence from future technology" (p.12).

Procedure:

The population of the study was the Jordanian students in the 9th grade who study some of the geometrical concepts, principles, and constructions. The sample of the study consisted of 52 students in the 9th grade at the Model School of Yarmouk University, Irbid, Jordan in the academic year 1999/2000. There were 26 students in the experimental group and 26 students in the control group. Both groups were being taught by the same teacher. The experimental group studied the geometrical part of the curriculum by using the book and the Geometer's Sketchpad (GSP) software, while the control group studied the same part using only the book. The students in the experimental group used the GSP once a week during the first semester of the academic year 1999/2000.

At the end of the experiment, all students in the sample took a test measuring their understanding of some of the geometrical concepts focusing on the relationship between the area and the perimeter of polygons such as rectangles and triangles (Appendix A). The instrument (achievement test) used in this study was designed by the researcher, and indicated that it was a valid one by some of the mathematics educators in Jordan.

There were four hypotheses in the study:

1. There is a significant difference between the means of the pretests and the posttests for the experimental group.
2. There is a significant difference between the means of the pretests and the posttests for the control group.
3. The mean of the pretest results for the experimental group is equal to that for the control group.
4. The mean of the posttest results for the experimental group is greater than that for the control group.

In order to study these hypotheses, the researcher found the means of the students' results in both groups on the pretest and the posttest, and used the ANCOVA test to compare and analyze the results.

The Results and Discussion:

This research was designed to study the effect of using the GSP on students' understanding of some of the geometrical concepts. The correlations between the means of the students' results on the pretest and the posttest of the control group, the experimental group, and the whole sample group were 0.966, 0.758, 0.681, respectively. According to Tables (1), (2), and (3), all of these results were significant at the 0.01 level (2-tailed). This result can be understood by looking carefully to the students' understanding of the geometrical concepts in both groups. The students in the control group did not gain more scores from the pretest to the posttest, which might be explained by using the regular way of teaching and learning, which is using only the book without using the computer. While the students in the experimental group gained more scores from the pretest to the post test, which refers to their use of computers and the GSP program. These results goes with the results of other studies (Dixon, 1996, and Yousef, 1997).

Correlations[a]

		PRETEST	POSTEST
Pearson Correlation	PRETEST	1.000	.966**
	POSTEST	.966**	1.000
Sig. (2-tailed)	PRETEST	.	.000
	POSTEST	.000	
N	PRETEST	26	26
	POSTEST	26	26

**. Correlation is significant at the 0.01 level (2-tailed).

a. group = control group

Table (1): The Correlations between the Pretest and the Posttest for the Control Group

Correlations[a]

		PRETEST	POSTEST
Pearson Correlation	PRETEST	1.000	.758**
	POSTEST	.758**	1.000
Sig. (2-tailed)	PRETEST	.	.000
	POSTEST	.000	.
N	PRETEST	26	26
	POSTEST	26	26

**. Correlation is significant at the 0.01 level (2-tailed).

a. group = experimental group

Table (2): The Correlations between the Pretest and the Posttest for the Experimental Group

Correlations

		PRETEST	POSTEST
Pearson Correlation	PRETEST	1.000	.681**
	POSTEST	.681**	1.000
Sig. (2-tailed)	PRETEST	.	.000
	POSTEST	.000	.
N	PRETEST	52	52
	POSTEST	52	52

**. Correlation is significant at the 0.01 level (2-tailed).

Table (3): The Correlations between the Pre-test and the Post-test for the Whole Sample Group

In Table (4), the results of the descriptive statistics indicated that the mean of the post-test results was 41.5385 for of the control group, while it was 68.6538 for the experimental group. Figure (1) also gives another perspective of this significant difference. In the same time, the results showed that the scores of the experimental group students on the post-test were more deviated than those of the control group. In addition, the standard deviation of the scores for the whole sample group was larger than that of both groups. This result can be understood by combining together all the scores of the tests of both groups.

Descriptive Statistics

	group	Mean	Std. Deviation	N
POSTEST	control group	41.5385	15.0844	26
	experimental group	68.6538	21.0978	26
	Total	55.0962	22.7409	52

Table (4): Descriptive Statistics for the Control and the Experimental Groups

Table (5) showed the results of the ANCOVA test. All F values in this study were significant under the 0.05 level. This result showed that the use of the GSP had the effect on students' understanding of the geometrical concepts, and other research studies support it. On the other hand, the pairwise comparisons showed that mean differences were significant under the 0.05 level, which can be considered as another evidence of the results of this study.

Tests of Between-Subjects Effects

Dependent Variable: POSTEST

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Noncent. Parameter	Observed Power[a]
Model	179105.161[b]	3	59701.720	571.382	.000	1714.146	1.000
PRETEST	11696.507	1	11696.507	111.943	.000	111.943	1.000
GROUP	10289.292	2	5144.646	49.237	.000	98.475	1.000
Error	5119.839	49	104.487				
Total	184225.000	52					

a. Computed using alpha = .05

b. R Squared = .972 (Adjusted R Squared = .971)

Table (5): ANCOVA Test for the Dependent Variable (Posttest)

Pairwise Comparisons

Dependent Variable: POSTEST

(I) group	(J) group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
control group	experimental group	-26.3329*	2.836	.000	-32.032	-20.634
experimental group	control group	26.3329*	2.836	.000	20.634	32.032

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

Table (6): Pairwise Comparisons

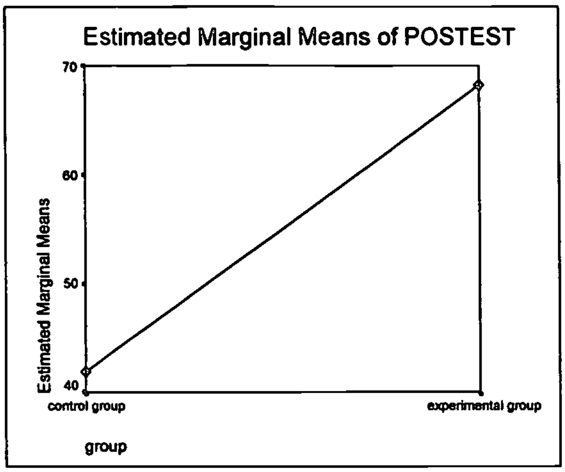

Figure (1): The Profile Plots

Recommendations:

According to the results of this study, the researcher had some suggestions and recommendations:

1. This study had the sample from students in the 9th grade. This means that there is a need for further studies in other grades and levels.
2. The sample of the study consisted only males. It is recommended to conduct other studies in the same area with samples from both males and females.
3. Since this study as well as other previous studies concluded that there was a significant effect of using the GSP software, the researcher recommends more emphasize on the use of computer and its programs in mathematics and in education.
4. The GSP is one of the latest computer programs in the mathematics area. It is recommended to evaluate its features and capabilities.

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References:

Becker, H. (1991). How Computers are used in United States Schools: Basic data from the 1989 I.E.A. Computers in Education Survey. *Journal of Educational Computing Research*, 7(4), 385-406.

Dixon, J. (1996). English Language Proficiency and Spatial Visualization in Middle School Students' Construction of the Concepts of Reflection and Rotation used the GSP. *Dissertation Abstract International*, DAI-A 56111, University of Florida.

Finzer, W. and Bennett, D. (1995). From Drawing to Construction with The Geometer's Sketchpad. *Mathematics Teacher*, V88, n5, p. 428 -431.

Giamatti, C. (1995). Conjectures in Geometry and the Geometer's Sketchpad. *Mathematics Teacher*, Vol. 88, No. 6, p. 456-458.

Grabe, m. & Grabe, C. (1998). *Integrating Technology for Meaningful Learning*. Boston, MA: Houghton Mifflin Company.

Growman, M. (1996). *Integrating Geometer's Sketchpad into a Geometry Course for Secondary Education Mathematics Majors*. Association of Small Computer users in Education (ASCUE) Summer Conference Proceedings, 29th, North Myrtle Beach, SC.

Jackiw, Nicholas. (1991). *The Geometer's Sketchpad*. Berkeley, California: Key Curriculum Press.

Lester, M. (1996). The Effects of the GSP Software on Achievement Knowledge of High School Geometry Students. *Dissertation Abstract International*, DAI-A 57106, University of San Francisco.

Mehligner, H. (1998). Schools Reform in the Information Age. In *Computers in Education*, Hirschehl, J.; and Bishop, D. (Eds.), Guilford, CT: Dushkin/ McGraw-Hill.

Opponheimer, T. (1997). *The Computer Delusion*. Atlantic Monthly, July, 260, 45-62.

Quarterly Educational Data. (1996). School Technology Research Results [On-Line], Available: http://qeddata.com/results.html.

Stoll, C. (1996). *Silicon Snake Oil: Second Thoughts on the Information Highway*. ISBN: 0385419945, Anchor.

Yousef, A. (1997). The Effect of the GSP on the Attitude toward Geometry of High School Students. *Dissertation Abstract International*, A 58105, Ohio University.

Jordan Experience with Computer Based Instruction in Teaching Calculus and Statistical Methods

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ABSTRACT

A Jordanian team was formed by UNESCO to produce two computer-based instruction, CBI, packages to teach Calculus and Statistical Methods. These packages were produced in the period 1992-1999. This article provides some information about the design, the specifications and the implementation of these packages. Moreover, it gives an idea about the plan of the University of Jordan to teach selected mathematical topics using Mathematica for the academic year 2000-2001.

Based on this experience, the paper provides a comparison between classical instruction method and the CBI. Moreover, we discuss the effects of CBI on reforming the teaching methods, instructor's role, student's role, evaluation procedure, and curricula.

Illustrative examples of the UNESCO packages (if time permits) will be presented. These examples may include individualized questions generated and corrected by Mathematica, timed quiz corrected with feedback if some thing went wrong, components of report writing, detailed computations shown to the student, animated graphics, introducing a statistical concept and a concept from calculus through Mathematica.

* *On sabbatical leave from the University of Jordan for the year 1999/2000*

The UNESCO Packages

In 1992, ROSTAS started the implementation of UNESCO/ROSTAS Project for "Improving Teaching of Mathematical Sciences at Arab Universities". The key specific objectives of this project was to improve the quality of teaching Calculus at the undergraduate level by designing new curricula that will combine theory, applications and computer investigations. A team from Jordan entrusted with this project has been able to develop during the period 1992-1993 a new non-traditional approach for Calculus teaching with emphasis placed on the use of computer software package "Mathematica" for instruction, computation, and symbolic manipulation. In 1994, the team was able to revise the Calculus materials that were prepared in 1993. The new material was divided into three modules Calculus I, II and III.

The material consists of two parts.

a) Three computer diskettes including the three modules and a "teacher's file" which contains prerequisite tests, evaluation sheets, exams, worksheets, instructor's files and "using Mathematica" file. Each course is organized in 43 one-hour lessons, such that each lesson is a complete entity with discussion material, evaluation sheets and test sheets.
b) Four volumes of printed material. The first three volumes include the use of Mathematica together with the three courses of Calculus. Volume 4 entitled Calculus Lab is divided into three parts each provides 14 Lab sessions for each course.

A member of the team was able to implement the new approach of Calculus learning using computer on a small group of 10 students at Jordan University for Women during the Summer Term of 1994. The experiment was repeated during the Spring Semester of 1995 on 35 students at Amman National University. Same experiment was implemented on 40 students at the Hashemite University in the Summer Term of 1997. The results are quite promising. The Mathematics and the Engineering Departments at Mu'ta University started using this package since 1993. The instructors and students there are very happy with it. The feedback of these teaching experiments was very fruitful in improving the package.

Some members of the team conducted training workshops of the package at Mu'ta University, Suez Canal University, Damascus University, and Alexandria University. The faculty members who attended these workshops accepted the idea and raised some useful comments that helped in improving the package.

Another two-year project with the support of ROSTAS for teaching Statistical Methods started in 1997. Before the preparation of this new package, the team consulted with some experts from both USA and England. One member of the team spent 10 days in England for this purpose. The output of this project is the production of a complete software-Based course (with corresponding printed material) providing:

1. Introduction file which concentrates on *using computer* and MATHEMATICA
2. Interactive text files of the covered topics
3. Searchable course glossary which links to the text files
4. Computer *lab-work* using symbolical, graphical, numerical, and statistical capabilities of Mathematica

5.Selected *data sets* to be used in the course work
6.Training on real *case study*
7.Training on *report writing*
8.On-line timed *quizzes* corrected and graded by computer together with additional *self-evaluation* interactive questions, and *individualized questions* to be corrected by Mathematica.
9.A specially designed *front-end interface* for problem solving , case studies and computer lab work
10.A file of needed *codes*, which are not available in Mathematica.
11. An instructor's *file* with answers to problems, tests, and labs.

Navigation of the course components can be done through the menu of the course package.

One member of the team has implemented this course at the Hashemite University in the summer term of 1998. The results were promising.

Design of a CBI Program

The essential components of most university courses are *lectures, tutorials, assignments, independent readings, and case studies.* The book by Halmos (1985) may be considered as one of the best resources that deal with classical learning and teaching of Mathematics. In this book he presented his own experience in learning and teaching Mathematics. We will not handle the raised issues by Halmos, since we will concentrate on using Information Technology (IT) in learning and teaching Mathematics.

Any change in an instructional environment will have both positive and negative effects. By reflecting on what happens when we make changes in our course, we should be able to find ways to maximize the positive effects and minimize the negative effects of these changes. So, we have considered the following questions before we have designed our CBI programs, (see e.g. Bodner, (1997);

1) Does the program teach skills that we value or those that our students have difficulty with?
2) Are the computers being used to do something that requires a computer, or those things we would use a computer to do?
3) Which parts of the course computer can illustrate?
4) Are the students using computer because they want to, or because they have to?
5) Does the program increase the students' understanding of course content or their familiarity with computers?
6) Do the students have difficulty navigating through a CBI program?
7) Do we need computer to deliver instruction or to aid instruction?
8) Which topics and activities that can be removed from traditional course, and which new topics and activities that should be added to the new course?
9) How IT can be used to individualize the learning and evaluation processes, and how immediate feedback should be accomplished during the early stages of a course?

Specifications of a Computer Aided Instruction Package

After we have answered the above questions, we moved to the problem of selecting suitable computer software. A good computer aided instruction package should provide learning environment (see e.g. Ramsden (1992)). It should have the following specifications:

1) Graphical representation has been steadily increasing. To make this presentation more effective, an educational package should be capable of

a) Producing colored graphs and all required plots,
b) Controlling the thickness of curves, sizes of points, and scale of the graph,
c) Combining different graphs on one screen,
d) Animating the produced graphs, rotating and translating three-dimensional graphs.

2) Illustrating some concepts and performing some algorithms may depend heavily on numerical computations. So, an educational package should

a) Have a reasonable accuracy of numerical computations,
b) Be able to perform all required calculations and manipulate lists of numbers or symbols.
c) Be able to show the steps involved in the computations of any procedure or algorithm,

3) Handling data is the base stone in statistical analyses, so an educational package should be able to

a) Allow the user to enter his own data regardless of the size of the data and the number of involved variables;
b) Allow the user to save his data in a file, call his data any time he wants and from any program which he wants to run, save the output of his computations, and edit his data.
c) Simulate data when need comes.
d) Provide the user (through a database) with real data from the field of his specialty and from his environment.
e) Modify the used data from any part of the package to study the effect of such modifications.

4) Other Considerations

a) A recommended package should have a *reasonable cost*. If the package may be used for one more educational purpose such as teaching and learning several courses then this will lower the overall cost.
b) Users of the package should find it *easy to handle*. If there is a familiar package to the users then this will enable them to start right away without a need for more time to get accustomed to the syntax of a new package.
c) The *syntax* of the package should be as close as possible to the mathematical and statistical syntax.
d) A package will be more useful if it allows the user to *write his own codes*, whenever he wants to demonstrate, explore, represent, or calculate some thing which is not available directly in the package. But writing codes

should be kept to the minimal. This will also allow the instructors to upgrade the package according to their needs.

e) A package should be able to handle text as well as other things. It should have a certain type of *editor*, which can exchange text, data, and codes with other software.

f) A package should be able to generate individualized questions and allow students to enter their answers and give them immediate feedback by correcting them and give right answers.

Based on the above specifications, we have selected Mathematica to be the environment for our CBI programs.

Plan of the University of Jordan

At the University of Jordan the main difficulties that we faced are

1) Some faculty members do not want to use the packages. This may be because they feel that computer will take their role, they do not know how to use computer, or because they feel that students may lose computational skills.
2) The number of students in the calculus course is very, it is about 1200 students per semester, and so it is not easy to schedule the computer labs for all these students. Moreover, it is not easy to facilitate sufficient number of PC's for these students.
3) The student should take calculus I course at the first semester of enrollment in the university, while some students may have no idea about computer and even how to use the keyboard.
4) The questions of how to conduct the course, how to evaluate the students, and which topics to be instructed using computer took a great deal of discussion among faculty members.

To overcome these difficulties, the Mathematics department at the University has decided to offer a new three-hour course called "Mathematical Computer Packages" for the forth year students. This course will introduce the main packages. One package will be selected and used to illustrate some selected concepts, discover some facts, build algorithms, do numeric and symbolic computations, do simulations, plot curves and draw graphs. The topics will be selected from Calculus, Linear Algebra, Differential Equations, Statistics, Vectors, Set Theory, Number Theory, Variational Methods, Graph Theory, Special Functions, Laplace and Fourier Transforms. It is expected that by the end of the course the student should be able to write his own program and prepare a term paper on one of the selected topics. More than one-faculty member will be involved in conducting the course. A training workshop will be conducted to the faculty members before the course starts.

Moreover, the universities in Jordan will start next year offering two obligatory courses in Computer Science for all students in their first year.

Comparison Between Usual Lecture Instruction and CBI

To convince our colleges on the benefits of using CBI, we have formed a discussion group to conduct a comparison between CBI and usual lecture instruction. We have agreed on the points that are given in the following table.

Component	**Usual lecture instruction**		**CBI**
Environment	*HW*	Classroom, blackboard, overhead projector, chalks…	PC with Multimedia
	SW	Instructor, students	Packages…
	Resources	Books, real labs, slides, transparences, …	Internet
	Note	some thing may not be clear	System may be down
Actors	-Students are less active -Instructor takes the complete role		-PC - Users are more active in watching what is happening
Time	-Limited time - Activities are limited by available time		-Unlimited time - More activities and drills may be generated
Case study	-Field work that opens the route for development of new techniques		-Simulation work that helps in illustrations but never match real word. -Discover evidence to support conjectures
Individualization	All students are treated at the same footings		Each student takes his own time according to his abilities
Flexibility	-Can change course of action according to responses, -Can switch to prerequisites and fill in the gaps -Student can find an immediate response to his questions		- Ridged to some extent - Remedial word is more difficult to implement - Student may not find an immediate answer to his questions
Interaction	-Students & Instructor - Corrections are supplied if students are not shy or afraid to respond and class size allows for enough responses		-PC & users - No reason for the student to be shy.
Feedback	-For student: Feedback on evaluation may come at a very late stage -For instructor: immediate		-For student: Can come immediately at each step if the package is programmed to do so. -For instructor: May come at a very late stage
Sound	-talking: Can't go back to it		-Sound plaster: Can go back to it
Text	-Writing on blackboard: Can't go back to it		-Editor: Can go back to it
Graphics	-Free hand drawing at most two dimensional -Neither neat nor accurate -Time consuming		-All types of graphics in several dimensions -Neat and accurate -Save time
Computations	-Hand calculations -Not accurate -Time consuming		-Numerical and symbolic calculations -Accurate -Save time
Illustrations	-Demonstrations		-Animation

Effects of CBI on the Instructional Environment

It also was agreed that the teaching and learning components might be affect by CBI in one or more of the following points:

1. Development of concepts numerically, graphically and analytically.
2. Saving time spent on lengthy computations and drawing graphs.
3. Producing accurate and nice graphical representations.
4. Simulating experiments and data from some probability models.
5. Simulating characteristics of probability distributions, properties of some statistics, and powers of tests where analytical results are not available
6. Using simulation to discover some facts and support some conjectures.
7. Animating diagrams to demonstrate and/or discover concepts and properties.
8. Providing the students with confidence in their work, when they compare what they obtained by hand with the computer solution,
9. Providing each student with his/her enough time and enough training to go through the course according to his/her ability.
10. Showing the students the power and the limitations of computer in order to get as much as possible of its power and to investigate the obtained results.
11. Using available related courses on the Internet.
12. Sparing the instructor's time to guide his students and suggest suitable activities.
13. Producing frequently asked questions' (FAQ) file together with answers.
14. Searching for relevant material on the Internet.
15. Sparing the student's time to concentrate on his ability to identify problems and possible solutions; collect data using an appropriate methodology; use a suitable software package to analyze data and interpret obtained results, recognize limitations and communicate the results in written and verbal form.
16. Providing students with activities according to their abilities and time.
17. Developing students skills on using statistical packages to prepare them for their subsequent employment, since these packages are widely used in research and work in the future.
18. CBI can be confusing for some students to investigate and, unless directed with care, a student can leave such a program without having observed all the important situations.
19. Regular self-testing of progress to maintain standards by showing the gaps in the student's knowledge.
20. Added topics in Statistics: Simulation, Bootstrap methods, …
    Removed topics in Statistics: Coding data,

Illustrative Examples

The following will be demonstrated (if time permits)

1. Individualized questions generated and corrected by Mathematica
2. Introducing the concept of the limit.
3. Illustrating the central limit theorem
4. Detailed computations shown to the student
5. Animated graphics
6. Timed quiz corrected with feedback if some thing went wrong
7. Components of report writing

References:

Altham, P.M.E (1998), *Using S-Plus for teaching generalized linear modeling*, Math&Stat, 9, 2, 17-20.

Bodner, G. M. (1997), *Confession of a modern Luddite: A critique of computer-based instruction*, CAL-laborate, July 1997,4-6.

Davis, G. (1997) *Teaching with Minitab and SPSS in a business school environment*, Math&Stat, 8, 3, 11-13.

Halmos, P. R. (1985) , I want to be a mathematician, Springer-Verlag, NY Hunt, N. (1996) *Teaching statistics with Excel 5.0*, Math&Stats

Ramsden, P. (1992), *Learning to teach in higher education*. London Routledge.

Young, K. (1995), *First Bayes*, Math&Stat.

The Potential Role of Artificial Intelligence Technology in Education.

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ABSTRACT

The field of Artificial Intelligence (AI) and Education has traditionally a technology-based focus, looking at the ways in which AI can be used in building intelligent educational software. In addition AI can also provide an excellent methodology for learning and reasoning from the human experiences. This paper presents the potential role of AI in the various aspects of education. Six AI fields are presented. The first presents the knowledge representation (KR) which includes ontologies; new concepts for representing, storing and accessing knowledge; and schemes for representing knowledge. The second is related to the use of Case-Based Reasoning (CBR) methodology in developing interactive intelligent educational systems for learning and teaching. The third topic is related to the use of Natural Language Processing (NLP) for analyzing the educational Web pages. The fourth is concerned with the Intelligent Tutoring Systems (ITSs), which are capable of adaptive instruction by means of multiple representations of domain knowledge. The fifth is the Intelligent Tutoring Systems Authoring Sheels (ITSASs), which allow a course instructor to easily enter domain and other knowledge without requiring computer programming skills. The last area deals with the learning in Distributed Artificial Intelligence (DAI). Moreover, the paper will explore a proposal for master degree in artificial intelligence in education

Introduction:

The field of AI in education has become the most challenging area in the last several years. It includes the disciplines; cognitive and social psychology, artificial intelligence, computer science, empirical psychology, software engineering and education. The goal of the field is to deliver computer-based systems (or knowledge-based software) which can be used in real teaching, learning and training situations.
The paper is structured as follows, section 2 presents a brief overview of the knowledge representation schemes and some learning concepts for representing episodic information. The importance of case-based reasoning methodology in teaching and training systems is discussed in section 3. Section 4 gives a brief discussion of the role of the natural language processing in the internet-based learning systems. Sections 5 and 6 are dedicated to the intelligent tutoring systems and authoring shells respectively. The area of learning in distributed artificial intelligence systems is discussed in section 7. In section 8, the paper presents a proposal for master degree in IA in education. Finally, section 9 summarizes the paper and raises some conclusions.

Knowledge Representation and Learning Concepts:

The two main parts of any AI software (or any AI-based educational software) are (1) a knowledge base and (2) an inferencing system. The knowledge base is made up of facts, concepts, theories, procedures and relationships representing real world knowledge about objects, places, events, people, etc. The inference system or thinking mechanism is a method of using the knowledge base, that is, reasoning with it to solve problems.

Knowledge Representation Schemes:

To build the knowledge base, a variety of knowledge representation schemes are used including logic, lists, trees, semantic networks, frames, scripts and production rules (See figure 1).

Declarative KR Schemes	Procedural KR Schemes
Are used to represent facts and assertions, e.g.	Deal with actions or procedures, e.g.
Logic	Subroutines
Semantic networks.	Production rules
Frames.	
Scripts.	

Figure 1: Knowledge Representation Schemes

Learning Concepts of Representing Episodic Knowledge:

Schank (1982) has advanced a number of concepts to deal with text analysis, human memory, and learning. Schank's concepts include: Script, Memory Organization Packets (MOP), and Reminding and Explanation Patterns (XPS). A brief overview of each of these concepts is presented in this section.

Scripts: Scripts accounted for information about stereotypical events, (e.g. going to a restaurant, taking a bus and visiting the dentist). In stereotypical event (common situations) a person has a set of expectations concerning the default setting, goals, props, and behaviors of the other people involved. Scripts are inherently episodic in origin and us, i.e., scripts arise from experience and are applied to understand new events. The acquisition of script is the result of repeated exposure to a given situation. (e.g. children learn the restaurant script by going to restaurant over and over again). As a psychological theory of memory, scripts suggested that people would remember an event in terms of its associated script.

Memory Organization Packets: MOP is a more general knowledge structure accounts for the diverse and heterogeneous nature of episodic knowledge. MOPs can be viewed as metascripts; e.g. instead of a dentist script or a doctor script, there might be a professional-office-visit MOP that can be instantiated and specified for both the doctor and dentist episodes. This MOP would contain a generic waiting room scene, thus providing the basis for confusion between doctor and dentist episodes.

Reminding and Explanation Patterns: Schank (1986) proposed a theory of learning based on reminding. The main features of this theory can be summarized in the following points.

(1) Conform-Driven Learning: When the new situations (or experiences) conform the past cases and events. Thus, we can classify a new episode in terms of past cases. The knowledge of the past case, like a script, can guide our behaviour.

(2) Failure-Driven Learning: When the new situations does not conform to the prior case, we have a failure. That is, we had an expectation based on a prior event that did not occur in the new situation. Thus, we must classify this new situation as different from the previous episode, we must remember this new experience and we must learn.

(3) Discrepancy-Driven Learning: When we observe a discrepancy between our predictions and some event. Thus, we have something to learn and we need to revise our knowledge structure.
The mechanism for updating our knowledge often requires explanation. Schank proposed an explicit knowledge structure, called "explanation patterns", that is used to generate, index, and test explanations in conjunction with an episodic memory.

Case-Based Reasoning (CBR):

The field of reasoning is very important for the development of AI-based educational software. The research area in this field covers a variety of topics (See Fig. 2). This section is dealing with the case-based reasoning which receives increasing attention within the AI in education community.

Automated reasoning.	Model-based reasoning.
Case-based reasoning.	Probabilistic reasoning.
Commonsense reasoning.	Causal reasoning.
Fuzzy reasoning.	Qualitative reasoning.
Geometric reasoning.	Spatial reasoning.
Nonmonotonic reasoning.	Temporal reasoning.

Figure 2: Reasoning Techniques

CBR is a general paradigm for reasoning from experience. It assumes a memory model for representing, indexing, and organizing past cases and a process model for retrieving and modifying old cases and assimilating new ones (Kolodner, 1993; Barletta, 1991; Salde, 1991). CBR has already been applied in a number of application areas, such as legal reasoning, dispute mediation, and customer support. A typical functional diagram of a CBR system is shown in Figure 3, adapted from Salde (1991). When a new problem is introduced in the system, the problem is indexed, and subsequently, the indexes are used to retrieve past cases from memory. These past cases lead to a set of prior solutions. Subsequently, the previous solutions are modified to adapt to the new situation. Then the proposed solution is tried out: If the solution succeeds, then it is stored as a working solution; if it fails, the working solution must be repaired and tested again.
CBR community have began to build CBR systems in education. (Kolodner, 1995). Examples of case-based educational and training systems are:
Schank's ASK systems (Ferguson, et al., 1992) take on the role of expert and guide a user dialog in which the system tells stories to make its points.
Archie-2 (Domeshek & Kolodner, 1993), is used in several architecture studios at Georgia Tech to help student designers with their projects.
Design Muse authoring tool (Domeshek, et al., 1994), is used in classes as well, both to build useful case libraries for several of engineering calsses and to give students the opportunity to learn more about some area by preparing and indexing well-articulated cases.
CBTS (Salem, 1997), is used for automatic generation of educational web pages for teaching sea creatures.
TAO is used for training Navy Tactical Action Officers in making effective tactical decision under battle conditions (AI @work, 1999).

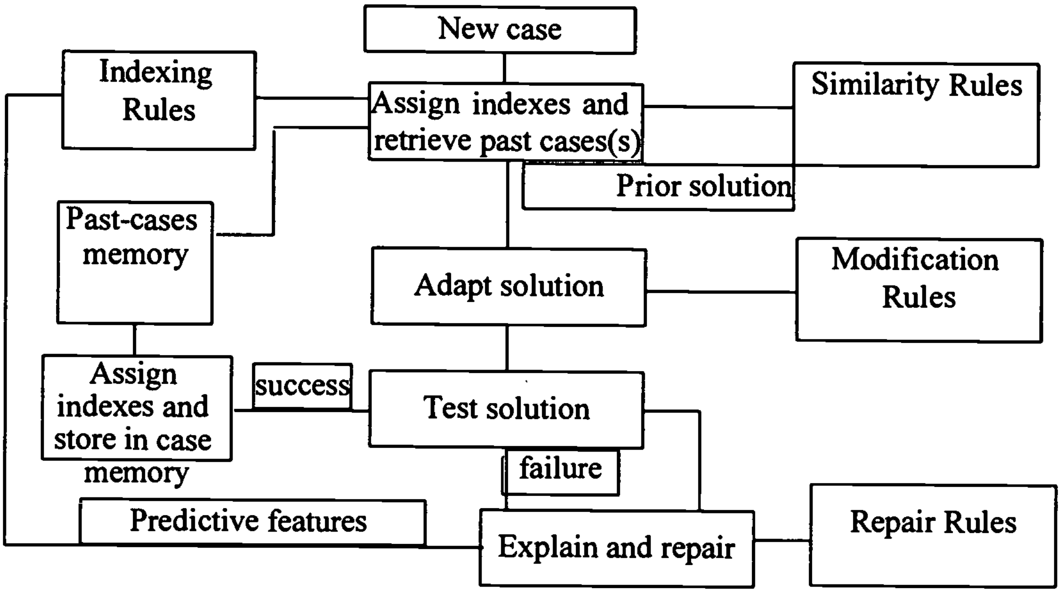

Figure 3. Case-based reasoning flow chart. Boxes represent processes and ovals knowledge structure.

Natural Language Processing (NLP):

NLP has become one of the most challenging area in the last several years. The research covers the fields: corpus-based methods, discourse methods, formal models, machine translation, natural language generation, natural language understanding, spoken language generation and spoken language understanding. NLP techniques, based on syntax and semantics of natural languages, play essential role in the internet-based learning systems and analyzing the educational web pages. The following research problems are very important:

- NLP for improved precision in document retrieval.
- Natural language interfaces for better document search.
- Semantic indexing of web pages.
- Summarization and information extraction.
- Machine translation of foreign-language web pages.
- Question answering on the web.
- Speech interfaces to web services.
- Dynamic creation of web pages using natural language generation techniques.
- The nature of the web for linguistic processing.
- Automatic creation of hypertext links between related passage

Intelligent Tutoring Systems (ITSs):

ITSs are smart computer tutors and typically have an expert model, student model, instructional module, and interface. ITS must organize their knowledge in a lesson-oriented manner. This organization must be dynamically adjusted by the system according to student models. Automatic generation of exercises and tests is an important feature of ITS. Additionally, ITS enhances instructor productivity, enabling them to cope with generating more complex training systems required to provide higher skill levels to today's high-tech trainees. It also provides tailored instruction and remediation, while allowing flexibility in teaching methods, achieving many of the same benefits as one-on-one instruction. ITS face the knowledge-acquisition difficulty, productivity of ITS development is determined by the efficiency of their knowledge methods/facilities.

ITS may have their greatest impact in complex or "enigmatic" domains Koedinger (1991). These are learning domains which typically use notation or other means of representation that do not promote problem solutions and which typically are not transparent in their use (e.g., geometric proofs). As computer programs are able to offer students alternative mental models for representing mathematical concepts, power and flexibility will be afforded the student in problem solving (ANGLE and OPERA are good examples). ANGLE (Koedinger, 1991) is a geometric proof tutor which emphasizes student use of schemas in addition to knowledge of rules. Students are encouraged to parse geometric diagrams into meaningful chunks, or Diagram Configuration schemas, which can assist them in planning their proofs. OPERA (Schwarz, 1992) is an environment in which students can investigate the properties of algebraic operators. Through the use of various schematic representations, students are able to form mental models of concepts such as mappings, functions, and structures of algebraic expressions.

Intelligent Tutoring Systems Authoring Shells (ITSASs):

An ITSAS has been developed that allows a course instructor to easily enter domain and other knowledge without requiring computer programming skills. The authoring shell automatically generates an ITS focusing on the specified knowledge. It also facilitates the entry of examples/exercises, including problem descriptions, solutions steps, and explanations. The examples may be in the form of scenarios or simulations. It allows organized entry of the course principles and the integration of multi-media courseware (developed with well-known authoring tools) which includes descriptions of the principles or motivational passages. In addition to course knowledge, the instructor specifies pedagogical knowledge (how best to teach a particular student), and student modeling knowledge (how to assess actions and determine mastery).

The most recent ITSAS are DIAG, RIDES-VIVIDS, XAIDA, REDEEM, EON, INTELLIGENT TUTOR, D3 TRAINER, CALAT, INTERBOOK, and PERSUADE (Redfield, 1998). Some tools were meant for select authors or students and others were designed for a wide set of authors. Some tools were designed to work with a limited area of domain expertise, and some were designed for a wide range of domains. Some tools had one main instructional strategy, but others had many. Each tool had their own way of representing the student's knowledge and understanding of the material being taught. Some tools generated instruction directly from domain knowledge. Some relied on pedagogical knowledge about the domain to create instruction. Some provided simulation environments for practice and exploration. For example, KONGZI (Lu et al., 1995) is an automatic generation tool and composed of two main systems: knowledge acquisition system and ITS generation system. Using KONGZI, Lu et al. (1995) developed three systems of domains: mineralogy, medicine and botany. The first system for detecting and processing faults of drills; the second one is for teaching diagnosis of heart diseases; and the third is a system of teaching botanical classification.

Distributed Artificial Intelligence (DAI):

DAI is concerned with the study and design of systems consisting of several interacting entities which are logically and often spatially distributed and in some sense can be called autonomous and intelligent. Two primary types of DAI systems can be distinguished (Weiβ, 1996): distributed problem solving and planning systems, where the emphasis is on task decomposition and solution synthesis and multi-agent systems, where the emphasis is on behavior coordination. Many DAI systems of both types have been described in the literature, differing from each other in the entities involved (e.g., with respect to their number, the number of goals pursuit by them, their degree of autonomy, and their perceptual, cognitive and effectual skills) as well as in the interactions between the entities (e.g., with respect to frequency, level and purpose). Examples of DAI systems are collections of communicating expert systems and teams of cooperating assembly robots.

DAI systems typically are very complex and hard to specify in their dynamics. It is therefore commonly agreed that they should be equipped with the ability to learn, that is, to automatically improve their future performance. Today the area of learning in DAI systems receives steadily increasing attention within both the DAI and the machine learning (ML) communities. Fig. (4) shows the main aspects of the ML from AI point of view.

Analogical learning.	Inductive learning.
Bayesian learning.	Inductive logic programming.
Case-based learning.	Multistrategy learning.
Computational learning theory.	Reinforcement learning.
Connectionist learning.	Speedup learning.
Constructive induction.	Scientific discovery.
Decision-tree learning.	Theory refinement.
Explanation based learning.	Unsupervised learning.

Fig. (4): Machine learning area from AI point of view

A Proposal For Master Degree in Artificial Intelligence in Education:

This section presents a proposal for Master degree in "AI in education". The suggested topics of the program is based on the analysis of the abstracts of about 600 papers of the last four world conferences on "Artificial Intelligence in Education (AI-ED" which held during the period 1993 - 1999. The program consists of core courses, specialization tracks, and thesis. The total duration of the program is two-year (4 semesters). The first two semesters for theoretical study (equivalent 10 courses / 5 courses per semester) in addition graduate seminar and practical training. Semesters 3 and 4 for project and writing a thesis.

- Core Courses: (9 credits; 3 courses to be selected):
- Knowledge representation and Knowledge-Based Technology.
- Natural Language Processing.
- Reasoning Methodologies.

- Learning and Planning.
- Specialisation Tracks
- (18 credits, 2 courses from each track and 1 additional course in a track of specialisation plus graduate Seminar and practical training).
- First Track: Intelligent Tutoring Systems and Authoring Sheels
- Principle/Tools for Instruction Design.
- Collaboration Tools and Techniques.
- Evaluation of Instruction Systems.
- Authoring Systems and Tutoring Shells.
- Second Track: Intelligent Interfaces
- Visual and Graphical Interfaces.
- Human Factors and Interface Design.
- Non-Standard and Innovative Interfaces.
- Natural Language Interfaces.
- Third Track: Teaching Strategies and Student Modelling
- Theories of Teaching and Motivation.
- Student Modelling and Cognitive Diagnosis.
- Social and Cultural Aspects of Learning.
- Cognitive Issues in Knowledge Acquisition and Evaluation.

Conclusion and Summary:

Knowledge representation technology presents a number of learning concepts and techniques for representing the episodic information.

CBR methodology extends the power of ITSs. Case-based intelligent tutoring systems help students to analyze and repair their solutions. Also ITSs can analyze the outcomes to provide an accounting of why the proposed type of solution succeeded or failed.

NLP can contribute significantly as an effective interface for stating hints to educational algorithms and explaining knowledge derived by the intelligent tutoring and training systems. Also, NLP techniques, based on the syntax and semantics of natural languages, could improve the efficiency of the internet-based learning systems on the web.

ITS have their greatest impact in enigmatic learning domains. These domains use notation or other means of representation that do not promote problem solutions and typically are not transparent in their use. (e. g. geometric proofs)

ITSAS allows easy development and maintenance of training systems which are based on pedagogically sound instructional strategies. The software is domain-independent and thus useful for creating a wide array of intelligent tutoring systems for a variety of domains.

DAI technology offers a useful opportunity to distribute training across multiple sites while dramatically reducing travel-related training costs.

Machine learning area focus on developing algorithms that allow computers to refine the knowledge they are given and to learn from experience. In addition ML techniques could improve the search and retrieval accuracy and efficiency of the educational web pages.

References:

AI@work (1999). "*Intelligent Tutoring System on the Internet: A Case-Based Approach*", PC IA, July / August.

Barletta, R. (1991). "*An Introduction to Case-Based Reasoning*" AI Expert (August), pp. 43-48.

Domeshek, E. and Kolodner, J. L. (1993). "*Using the Points of Large Cases*". Artificial Intelligence for Engineering Design, Analysis and Manufacturing (AIEDAM), Bol. 7. No. 2, pp. 87-96.

Domeshek, E., Kolodner, J. L., & Zimring, C. (1994). "*The Design of a Tool Kit for Case-Based Design Aids*". Gero, J. S. & Sudweeks, F. (Eds.), Artificial Intelligence in Design 94, Kluwer Academic Publishers, Netherlands, pp. 109-126.

Ferguson, W., Bareiss, R., Birnbaum, L., & Osgood, R. (1992). "*ASK Systems.: An Approach to the Realization of Story-based Teachers*". Journal of the Learning Sciences 2: 95-134.

Koedinger, K. (1991). "*On the Design of novel Notations and Actions to Facilitate Thinking and Learning*". In L. Birnbaum (Ed.), Proceedings of the International Conference on the Learning Science (pp. 266-273). Charlottesville, VA: Association for the Advancement of Computing in Education.

Kolodner, J. (1993). "*Case-Based Reasoning*", Morgan Kaufmann, San Mateo.

Kolodner, J. (1995). "*From Case-Based Reasoning to Scaffolded Electronic Notebooks; A Journey*" Proceedings of the seventh AI in Education Conference, Washington, DC. USA (pp. 25 - 35).

Lu, R., Cao, C. Chen, Y., and Han, Z. (1995). "*On Automatic Generation of Intelligent Tutoring Systems*", Proceeding of the Int. Conf. Of Artificial Intelligence in Education, Washington D.C., USA, 67-74.

Redfield, C. L. (1998). "*Intelligent Tutoring System Authoring Tools*", AI Magazine, Vol. 19, No. 2, p. 126.

Salde, S. (1991). "*Case-Based Reasoning: A Research Paradigm*", AI Magazine, Vol. 12, No. 1, 42-55.

Salem, A. M. (1997). "*Educational Knowledge-Bases on the Web*", Proceedings of the Sixth EDEN Annual Conference on Open and Distance Learning, Budapest, Hungary. (pp 382 - 386).

Schank, R. C. (1982). "*Dynamic Memory: A theory of learning in Computer and People*", Cambridge University, Press, New York.

Schank, R. C. (1986). "*Explanation Patterns: Understanding Mechanically and Creatively*", Lawrence Erlbaum Associates, Hillsdale, New Jersey.

Schwarz, B. B. (1992). "*Advantages in the use of causal models for the learning of mathematical concepts*", ITS'92 Montreal, Canada.

Weiβ, G. (1996). "*Learning in Distributed Artificial Intelligence Systems*", Proceedings of the workshop, 12th European Conference of Artificial Intelligence, W26 Budapest, Hungary.

Impact of Using CAS in the Teaching of Mathematics

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ABSTRACT:

Recently there has been a great deal of discussion about students playing an active role in the learning process for they learn best if they experiment and construct their own forms of knowledge. In this presentation, we demonstrate how the advent of Computer Algebra Systems (e.g. Mathematica, MapleV,...) may help in supporting these ideas. If used properly, such systems can have a great impact on what we teach and how we teach and on students' learning of concepts and problem solving skills.

Introduction:

This paper is an outcome of the presentation given at the ICTCME meeting held at LAU, Beirut, Lebanon, July 4-6, 2000. It discusses the impact of using computer algebra systems (CASs) in the teaching of mathematics. The balance of this paper is as follows:

1. General Remarks
2. Impact of Using Computer Algebra Systems
3. Examples from: Calculus; Linear Algebra; and Differential Equations
4. Reflections

General Remarks:

We review some background on the status of the mathematics curriculum and departments with the intent of motivating the need to change and the infusion of technology in mathematics instructions. It is a common knowledge that the mathematics departments are suffering from a decline in the number of majors, students are not performing at the expected level and other disciplines we service are demanding an integration of their areas when we teach mathematics courses required by their students. These forces along with the advent of technology in the last 20 years necessitate the need to rethink the way we teach, what to teach and how students learn. I will discuss briefly each of these forces

Students' Performance There is enough data available that documents the increase in drop rates, the low retention and the inability of students to transform and integrate material learned in subsequent courses. The calculus sequence is the classic example of the "filter rather than a pump" scenario where we lose the students. The recommendations made by many professional organizations all emphasize that students need to play a more active role in the learning process. Passive learning with the instructor being the source and the student being the sink is not applicable and definitely lead to inadequate students' performance.

Majors It is rather unfortunate that mathematics departments on the whole are suffering from low enrollment. Fewer students are choosing mathematics as a major. It is equally unfortunate that the mathematics departments are becoming more of service departments than ever. Again, this is a major force that requires us to rethink the way we do our business.

Needs of Other Disciplines The curriculum is a dynamic system. Changes must be incorporated with the changing needs of the time. Engineering, Physical and Social Sciences departments are requiring from their majors not only to learn the skills and algorithms for solving a problem but also the applications and the interpretations are equally important. Furthermore, they want their majors to become familiar with the available technology to assist them in data analysis and massive computations. They want their majors to write meaningful reports. Since we in the mathematics departments are essentially serving the needs of other disciplines, it is a matter of survival to rethink the way we deliver and to incorporate into our curriculum such fundamental changes.

Such forces led the mathematical community to rethink the curriculum and the methods of delivery. Technology, on the other hand, developed to the point where it can have, if used responsibly, an impact on restructuring as well as the methods of teaching. In a nutshell, technology (calculators/computers) will furnish graphical representations and animations that can not be generated independently; facilitate organizing and analyzing data; compute efficiently and accurately; support investigation and exploration; facilitate experimentation; facilitate understanding of abstract notions; and support self paced learning.

The role of technology is beautifully summarized in the following quote:

> "If I can give an abstract proof of something, I'm reasonably happy. But if I can get a concrete, computational proof and actually produce numbers, I'm much happier. I'm rather an addict of doing things on the computer, because that gives you an explicit criterion of what's going on. I have a visual way of thinking, and I'm happy if I can see a picture of what I'm working with."

<u>Impact of Using Computer Algebra Systems</u>:

A computer algebra system (CAS) is a system capable of performing symbolic, graphical and numerical representations. Such systems are also well adapted for interactive learning. Examples of such systems include: MapleV, Derive, Mathematica, MathLab, MathCad, Mathwright and others. I will discuss the following points in which a CAS may impact the curriculum and delivery methods. The examples presented during the talk employed the computer algebra system MapleV. A partial list will be given in section 3 of this paper. These can be obtained via email.

1. Redefine what is important in mathematics

When a student claims that I know differentiation, is the student referring to the slope of the tangent line or the limit of the average rate of change? Or does the student mean that he can differentiate few functions and get an answer in closed form. Similarly, when a student says I know integration, is the student referring to the convergence of the lower/upper sums or the relationship between the anti-derivative of a function and the area under the graph? Or does the student mean I can evaluate integrals correctly in closed form. It is important to know if the student has mastered the meaning of the underlying concepts rather than the ability to manipulate. With the advent of CASs, routine algorithms can be handled using such packages and the emphasis will shift to the understanding of the concepts. Computations should not be the end but rather a means. The time saved should be devoted to concentrate on motivation, concept learning, applications and interpretations. Students develop self-confidence when they master and apply a concept rather than being "computational robots".

Therefore, a CAS becomes an effective tool for convincing students that the appropriate

focus should be the concepts rather than a mastery of few computational skills.

2. Role of Approximation and Error Analysis

It is rarely the case that "real applied problems" possess closed form solutions. Closed form solutions become the exception rather than the rule. As such, approximation and error analysis become essential parts of the curriculum. This topic is barely considered in the curriculum due to the tedious calculations involved. With the use of a CAS, this important topic can become an integral part of the curriculum. Numerical integration can become the norm and closed form integration as special case. Therefore, CAS becomes now an effective tool for shifting the emphasis from closed form solutions to solve open-ended problems and seek approximate solutions with apriori estimates.

3. Integration of symbolic, numeric and graphic representations

This is a great feature that definitely enhances problem solving techniques, sharpen the

analytical skills of students and encourages experimentation. It provides the user with a

great insight into problem solving. Indeed, use of a CAS changes the way we learn and do mathematics. A worksheet or a notebook developed using a CAS becomes an interactive mathematics document. Students can now learn in an interactive way, can become active participant in the learning process and can develop their own constructs. Learning in this mode can have a lasting impact.

4. New positive attitude towards problem solving

CASs enhance the problem solving skills by giving the user alternatives to: experiment, conjecture, tests and analyze the results. Change in attitude, therefore, is a major aspect of problem solving. Shifting the burden of computation to a CAS makes time available for students to concentrate on how to approach a problem, delineate sub-problems, consider alternatives and experiment.

5. Development of experimental approach to learning mathematics

We do not normally engage students in activities that require math explorations and experimentation. We consider students as recipients of a fixed body of facts and algorithms. We ask then to solve a non-homogeneous second order differential equation with constant coefficients but we do not ask about the behavior of the solution. We avoid variable coefficients equations or nonlinear problems. We ask to test for convergence but seldom we ask questions about the sum or the rate of convergence. On the other hand, CASs encourage students to develop an exploratory approach to learning and doing mathematics with focus on interpretations, concepts and ideas rather than on computations.

6. Assignments

Most of the exercises that we normally assign are drill type exercises. With the use of a CAS, there is an opportunity now to ask: open-ended problems; approximation of functions; error bounds; vector fields; phase portraits; approximate solutions of DEs; study of dynamical systems (discrete and continuous); large linear systems that model real situations; and so on. Indeed, we can augment our courses with well-structured labs as in the physical sciences to experiment and report the findings. We can now challenge the students using more practical problems and exercises.

Examples:

The examples that were demonstrated include: **Area** (this example demonstrates how by increasing the number of partitions the approximate area under the graph of a function gets closer to the exact area. Also, students will compare: right end point, Left end point, and mid-point approximations. Students will be asked to experiment and finally interpret the results and their findings. **Fresnel Integral** (this example demonstrates the importance of error bounds and approximation in computing this important integral arising in optics). **Bezier Curves** (this example demonstrates an area of geometric design. Students have the opportunity to generate all type of computer graphics and shapes. Again, such examples and projects are possible only upon using a CAS). **Critical points of polynomials** (this example demonstrates the importance of experimentation and exploration to determine the locus of the critical points of a polynomial by varying the coefficients one at a time). Other examples were also presented from **Cryptography** and **Differential Equations**.

Reflections:

My experience using a CAS in the teaching of mathematics has been very rewarding. Not only it helped me in teaching but also it enhanced my research productivity. I will not do it differently. The road for achieving some level of competency was rough but it is worth it. Using a CAS makes one think more about the mathematics than is usually conveyed. It makes one aware of the subtle points and the hierarchy of the thinking process. Students are enthusiastic and are very engaged in the learning process. They get immediate feedback for a query or a conjecture. Students, with some guidance, can work on challenging practical problems. For those of us who have an undergraduate research program, CASs are great tools for exploration. The learning environment is transformed from a passive one to an active one. Students' attitudes have changed. They are now positive and more engaged especially when they sense the utility of what they study. Fewer students are dropping now. We have attracted more majors in our degree program. For the process to succeed, it has to be integrated skillfully and is not enough to be an add on type activity.

An Exploration Of Mathematical Qualities Of Tasks Via The Use Of Technology

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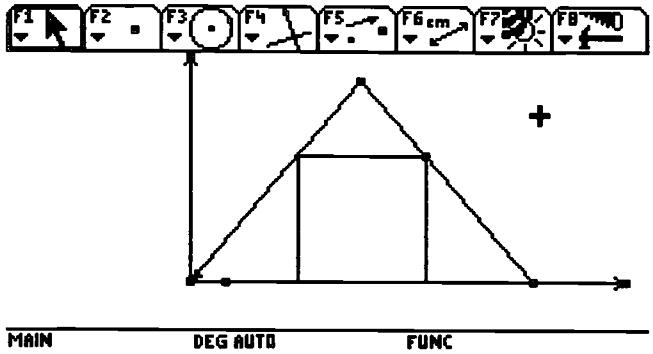
Recent mathematics curriculum reforms have pointed out the relevance of using technology in the learning of mathematics. Indeed, the use of graphic calculator or particular software such as dynamic geometry has produced changes not only in the type of tasks and questions that students examine during their solution processes; but also in the role played by both teachers and students throughout the development of the class. The National Council of Teachers of Mathematics (2000) identifies the use of technology as one of the key organizer principles of Pre-K-12 curriculum. How can we design learning activities in which the use of technology help or enhance the study of mathematics? What is the role of teachers/instructors in an enhanced technology class? To what extent mathematical arguments or ways to approach problems vary from traditional approaches (paper and pencil). These are fundamental questions that are used to frame and discuss four different approaches to the use of technology in the learning of mathematics. In particular, we document the use of the TI-92 Plus calculator and cabri geometry software as a means to work on nonroutine problems, to make and explore conjectures, to determine general patterns of recursive functions, and to compare differences and similarities between pencil and paper approaches and calculator or software approaches. In addition, it is shown that the use of this type of calculator helps visualize the problem or phenomenon from various representations that include the use of table, graphs and algebraic forms. These representations become important for students to identify and examine diverse mathematical qualities attached to the solution process.

(i) Access basic mathematical resources to work and solve non-routine problems.

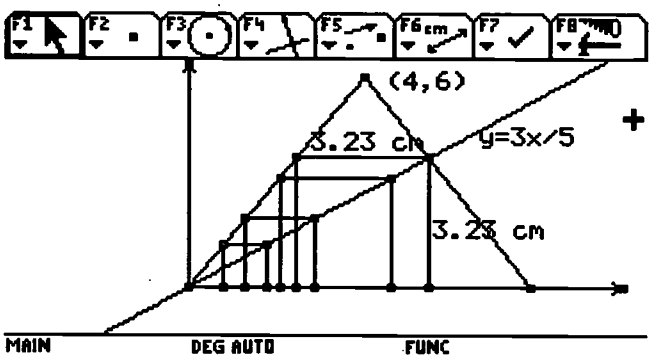
Several non-routine problems that require powerful mathematical resources for their solution with traditional approaches can become accessible to a variety of students. We have utilized the TI92 as a means to examine particular or simpler cases associated with various problems. This process leads students to eventually access basic resources that help them to approach them. To illustrate this idea we select a problem suggested by Polya (1945) and later used by Schoenfeld (1985) in his problem solving course. Although the essence of solution process rests on the use of a particular strategy (relaxing the original conditions of the problem and examining particular cases), it is clear that the use of technology offers advantages to visualize and evaluate cases that eventually lead to the solution.

> Inscribe a square in a given triangle. Two vertices of the square should be on the base of the triangle, the other vertices of the square on the two other sides of the triangle, one on each.



This problem was used in a problem solving session with high school students. The students' initial approach was framed through an open discussion of the following questions:

What is the given information? What does it mean to have a triangle? Does it mean that it is possible to know its vertices, or the length of its sides and the measure of its angles? What is the question or task? What are the conditions? Is it possible to quantify properties (areas, perimeters, slopes) of the shown figures and observe a particular relationship? Eventually, students suggested that an important problem solving strategy that can be useful to consider when the problem involves various conditions is to reduce them and explore their behavior through the analysis of particular cases (Polya, 1945). For instance, in this problem a key condition is that the four vertices of the square should be on the perimeter of the triangle. Hence, we can think of drawing a set of squares with only three vertices on the perimeter as the figure below:



The figure shows three squares with three vertices on the perimeter and the fourth seems to rest on a line. Indeed, cabri geometry can check easily whether the points are collinear. It is also easy to verify that the intersection of this line with the side of the triangle is the fourth required vertex.

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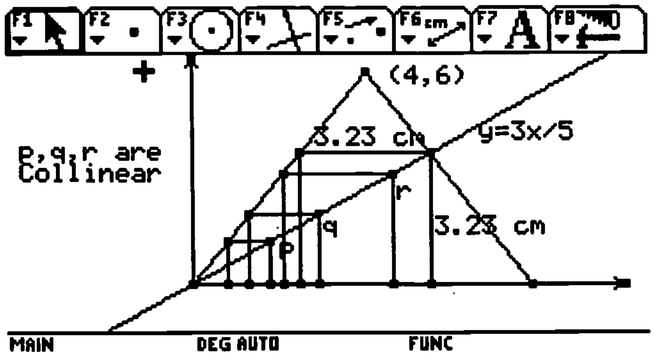
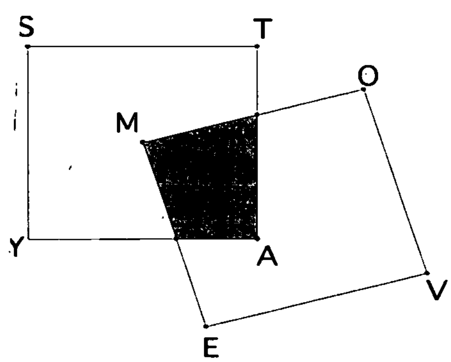


An important question here is: Is it possible to inscribe a square in any given triangle? Although the example shown above only represents a particular case, it is important to observe that it is possible to move vertex (4, 6) on the plane and check that for all the new triangles, that appear while moving such vertex, will maintain the inscribed square. That is, point p, q, & r will be collinear and the intersection of the line pq with the triangle side becomes the fourth vertex of the inscribed square. Indeed, these ideas can be applied to treat the general case in which (a_1, b_1); (a_2, b_2) & (a_3, b_3) are the vertices of the given triangle.
Students might also check the collinear property of points p, q, and r by using the distance criterion or slope condition. Is pq + qr = pq? Or is the slope of pq equal to the slope of qr.

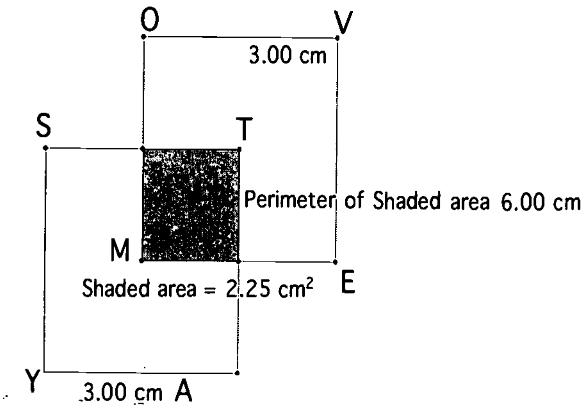
(ii) Find and explore different conjectures. The use of technology offers great potential for students to search for invariants and propose corresponding conjectures. Here, we illustrate an example in which students build dynamic environments to represent problems that eventually lead them to propose conjectures. The software becomes a tool for students to look and document the behavior of objects and relationships.

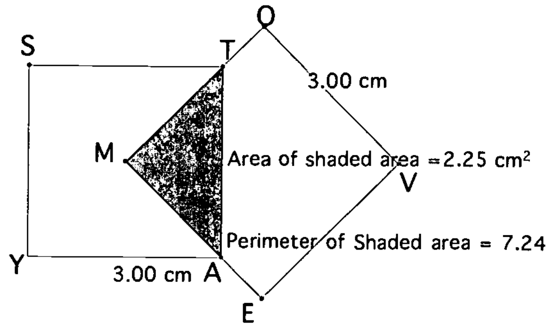
(a) Two congruent squares are placed so that vertex M of square MOVE lies at the center of square STAY. Square MOVE can rotate freely as it pivots about point M. When is the area of the shaded region the largest? Explain clearly and completely.
By using Cabri Geometry it is possible to construct both the fixed and mobile squares.



Here, students might discuss the information given in the problem and pose particular questions. The software allows students to explore cases in which they can quantify and record the value of area or perimeter of the common region.

How does the intersecting area behave when square MOVE rotates around its vertex M (Center of the fixed square)? How can we calculate the intersecting area? An important strategy that might be useful to consider here is the exploration of particular cases. For example, the figures below represent two cases in which the intersecting area is 1/4 of the area of one square. Indeed, these two cases include the analysis of 8 special cases in which the shaded area is always 1/4 of the area of one square.

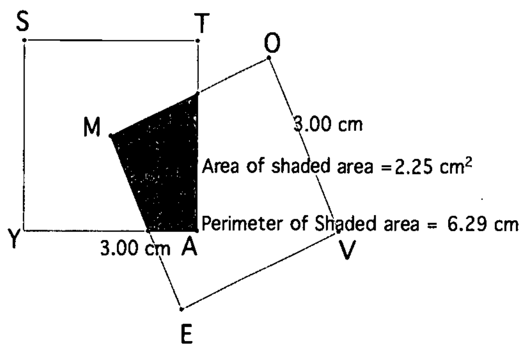




Points M, A, & E are collinear

The software also shows that for the case shown below (figure 3) the value of the intersecting area remains constant. Here, a conjecture may be established "the intersecting area of the two square is always 1/4 of the area of one of the square". To prove this conjecture, we examine the symmetry of the figure by proving that triangles KMW & RMV are congruent (figure 4.).

Figure 3.



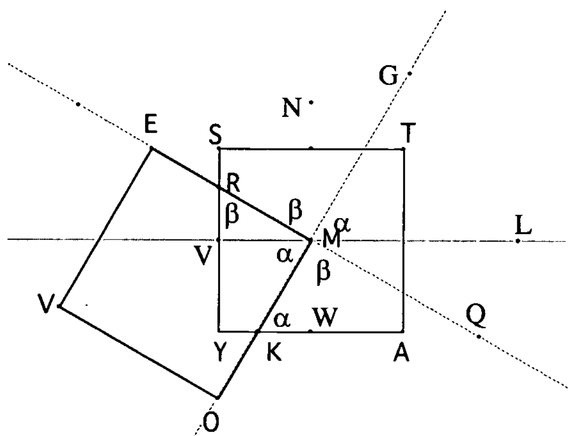
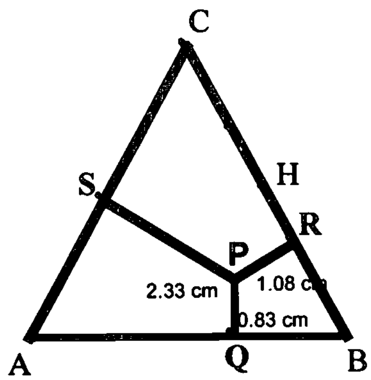


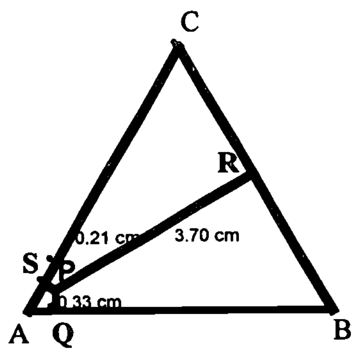
Figure 4.

Other related questions that emerged from the interaction with this problem included: How does the perimeter of the intersecting region behaves? When does it reach its maximum/minimum value?

(b) Given an equilateral triangle ABC and a random point P is placed inside of that triangle. If from P a perpendicular segment is drawn to each side of that triangle, then explore the behavior of the sum of the three perpendicular segments.

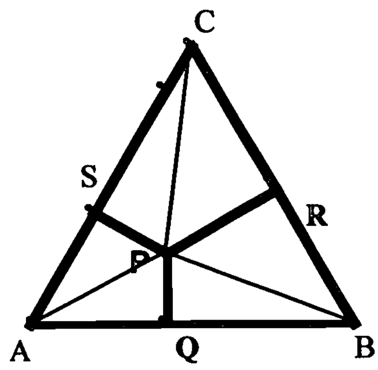


Result: PQ+PR+PS= 4.24 cm



Result: PQ+PR+PS= 4.24 cm

A conjecture that emerges from moving P inside triangle ABC is that the sum of the perpendicular segments drawn to each side is equal to the length of the altitude of the triangle. In order to prove this conjecture we draw segments from P to each vertex of triangle ABC (figure below). Here, three triangles can be identified: ΔAPC, ΔAPB & ΔBPC (figure below). Here, the area for each of these triangles can be expressed by (AC)(PS)/2; (AB)(PQ)/2 and (BC)(PR)/2 respectively. Since the triangle is equilateral then AC = AB = BC = l. The total area of the original triangle can be expressed by (l)h/2, where h is the altitude of the triangle. It is observed that (l)(h)/2 = l(PS + PQ + PR)/2. Here we have that h = PS + PQ + PR.



(iii) Generalize mathematical patterns using defined and recursive functions. An important goal during learning mathematics is that students develop different strategies to find and analyze the behavior of mathematical relationships that emerge from the consideration of particular cases. For example, we illustrate that the TI-92 can be used to identify a formula or general expression for data that represent a recursive phenomenon (evolution of a medical treatment).

Treatment Description: The patient will take some tablets and receives the following information:

i. Doses or amount of active substance of the tablet: 16 units

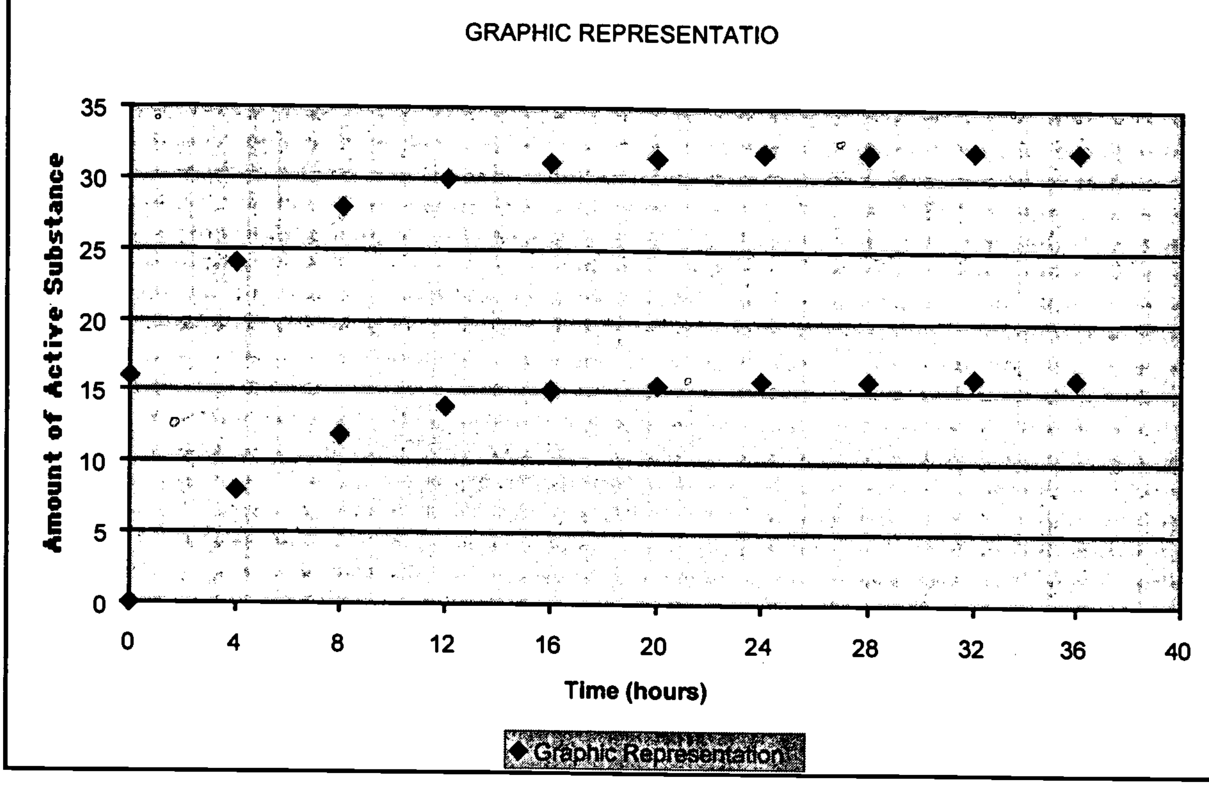
ii. When the patient takes the tablet, his organism begins to assimilate the active substance and 10 minutes after; his body will assimilate the total amount of 16 units. That is the patient organism will assimilate the doses 10 minutes after he took the tablet.

iii. When the patient organism assimilates the total doses of the tablet, his organism begins to eliminate the medication. The patient must take the next tablet when the previous doses are reduced to a half of that amount. The physician tells the patient that this reduction takes place every four hours. Thus, the patient will take the second tablet when the previous amount (16 units) assimilated from the first tablet is reduced to 8 units. This doses reduction will occur four hours later since the patient takes the first tablet. Thus when the patient registers 8 units, he will take the second tablet and his body will assimilate again the 16 units ten minutes later, here the amount of units stored in his body will be 8 + 16. Here, again his body commences the elimination period and when it reaches half of this amount (12 units after four hours), then the third supply takes place, and so on.

iv. The patient will follow this treatment during a week.

Based on the treatment description there is interest to analyze the amount of medication stored by the patient organism at different stages or moments of the treatment. In particular, What amount of medication is stored by the patient when he takes each tablet/ and 10 minutes later? Is it possible to discuss the evolution of the patient treatment in terms of mathematical resources? In order to respond the above questions, it will be important to identify relevant information associated with the treatment. For example, data that seem to be relevant include the amount of active substance of the tablet (16 units), the time in which the patient assimilates the active substance (10 minutes); frequency of dosage, amount of active eliminated by the patient during each drug admission.

Use of a table. There are different means to analyze the relevant information of the situation. For example, a table can be used identify possible relationships between data given in the situation. An important aspect here is to determine the entries which can help display the behavior of the relevant information during a period of time. Thus a table that includes frequency of dosage, total time (hours) at each take, amount of active substance ten minutes after each supply.



Supply # (every four hours	Elapsed time (hours)	Amount of active substance at each supply	Amount of active substance 10 minutes after each supply
1	0	0	16
2	4	8	24
3	8	12	28
4	12	14	30
5	16	15	31
6	20	15.5	31.5
7	24	15.75	31.75
8	28	15.875	31.875
9	32	15.9375	31.9375
10	36	15.96875	31.96875

Describe the amount of active substance stored in the patient's body at the moment of each supply and ten minutes after.

How much time has passed after the nth supply?

(b) Graphic Representation. By using the data shown in the table, it is possible to present a visual approach or graphic representation of relevant information.

What differences do you see between data represented in the table and data displayed in the graphic representation? It is observed that both representations show that the amount of active substance at each taking and 10 minutes after converge to 16 and 32 respectively.

(c) An Algebraic Approach. Is it possible to analyze regularities observed in the two above representations via the use of algebra? Here, the information of the table will be used to examine the pattern.

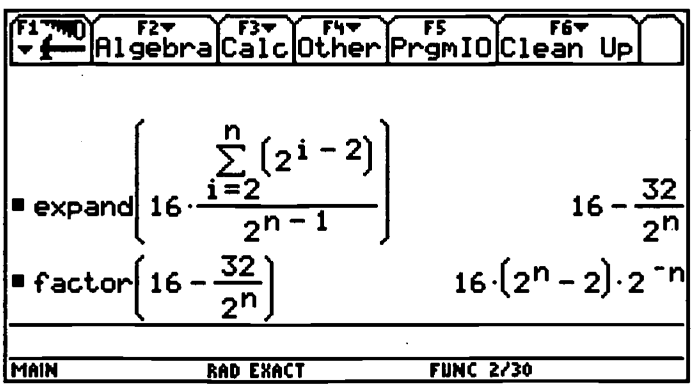
Supply No.	Amount of active substance stored in the patient's body at each supply
1	0
2	$\frac{0+16}{2}=\frac{16}{2}$
3	$\frac{\frac{16}{2}+16}{2}=\frac{16+2\times 16}{2^2}$
4	$\frac{\frac{16+2\times 16}{2^2}+16}{2}=\frac{16+2\times 16+2^2\times 16}{2^3}$
5	$\frac{\frac{16+2\times 16+2^2\times 16}{2^3}+16}{2}=\frac{16+2\times 16+2^2\times 16+2^3\times 16}{2^4}$
6	$\frac{\frac{16+2\times 16+2^2\times 16+2^3\times 16}{2^4}+16}{2}=\frac{16+2\times 16+2^2\times 16+2^3\times 16+2^4\times 16}{2^5}$

$$Cn=\frac{16+2\times 16+2^2\times 16+2^3\times 16+\cdots+2^{n-2}\times 16}{2^{n-1}}=$$

$$=16\left(\frac{1+2+2^2+2^3+\cdots+2^{n-2}}{2^{n-1}}\right)$$

$$=16\left(1-\frac{1}{2^{n-1}}\right)$$

The above formula can be obtained via the TI-92. Here it is important to introduce the general expression Cn in a condensed form. By using the commands expand and factor from the Algebra menu, the general formula is easily obtained.



The expression that describes the amount of active substance stored by the patient 10 after each supply is:

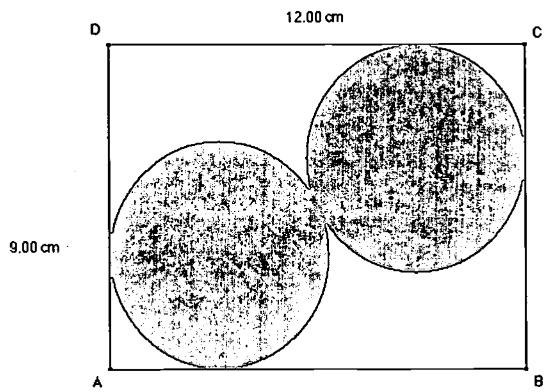
$$A_n = C_n + 16 = 16\left(1 - \frac{1}{2^{n-1}}\right) + 16 = 16\left(2 - \frac{1}{2^{n-1}}\right)$$

Relevant Data	Particular Situation	General Case
Amount of active substance or doses of medication (tablet)	16 units	d
Factor of elimination	1/2	r
Period of assimilation	10 minutes	t
Time between each supply	4 hours	s

Students can transform the original information given in the situation into general data. For instance, they can represent the amount of medication, factor of elimination, period of assimilation and time between each supply by d, r, t, and s respectively. Then, they can find a general expression for the amount of medication assimilated by the patient at each supply.

(iv) Compare differences and similarities between pencil and paper approaches and calculator approaches. . Here, we show that there are events or situations in which the objects and assumptions used to support results that emerged from the use of technology are different from those used in traditional geometry. As Goldenberg and Cuocuo (1998) stated "what is quite clear is that geometry on a computer is different from geometry on paper" (p. 365).

Two congruent circles are to be cut from a 9 cm x 12-cm sheet of construction paper. What is the maximum possible radius, to the nearest hundredth of a centimeter, of these circles? What percent, to the nearest tenth, of the area of the sheet will be cut? (*Calendar*, November 1997, #22). See figure 1



Exploration of relationships via auxiliary constructions: We draw line EF that passes through T and divides the rectangle in two congruent figures (see figure 3(a).

Based on symmetry the problem is reduced to finding the circle of the greatest area inscribed in one of the drawn figures. Ray EF meets ray AD at G (extending rays EF and AD). Here we identify triangle AEG and ask ourselves: How can we inscribe a circle in this triangle? We draw the angle bisectors in this triangle to locate the center of the inscribed circle (intersection of the bisectors). We draw a perpendicular segment from the center to any side of the triangle AEG to find the radius of the circle, which does not necessarily pass through T. The radius is less than the distance from the center of the circle to point T by the Pythagorean Theorem. This is because OT is the hypotenuse of right triangle OTM.

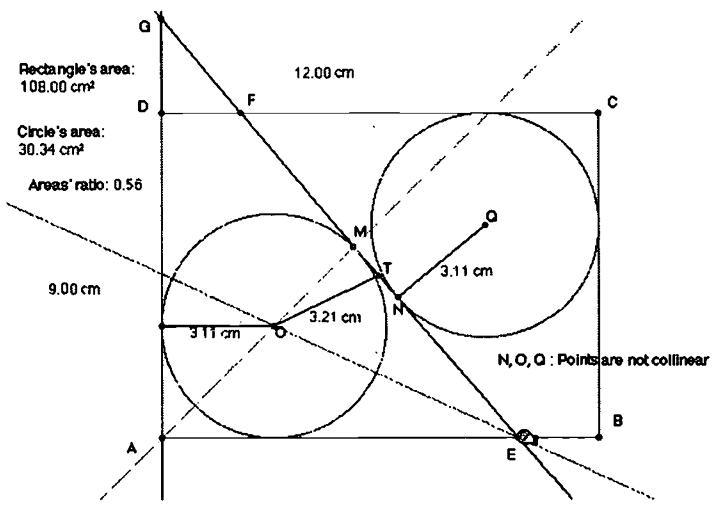
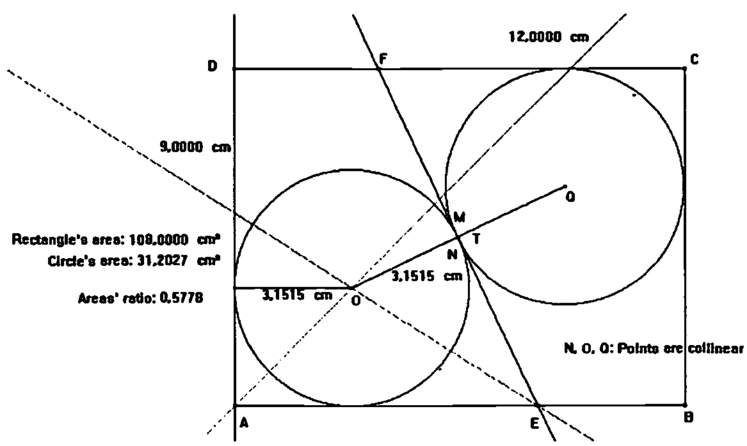


Figure 3(a)

Now we change the inclination of line EF by moving E, and observe that the radius of the inscribed circle will increase to reach the value OT. A similar construction on the other quadrilateral EBCF shows that the maximum value of the radius will occur when both centers of the circles and point T are on the same line. Thus, the collinearity property becomes a key factor to finding the solution.



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Final Remarks. Different aspects of the mathematical practice are displayed while using technology to approach problems. In particular, representing data seems to be an important step for accessing basic mathematical resources that become crucial to approach the tasks. Working with dynamic environments helps students identify and examine particular conjectures. It is also shown that the process of working with the problems via technology introduces a natural environment for posing and pursuing related questions. Indeed, seeing different dynamic representations of the problems helps students visualize connections and realize other approaches to the problem (Santos & Díaz Barriga; in press). It is also evident that even when the problems or tasks may not call for the use of the computer directly, the software could become a vehicle for examining their mathematical qualities. As a consequence, students get engaged in the process of utilizing diverse representation as a means to approach the problems and explore other related questions.

References

Courant, R. & Robbins, H. (1996). What is mathematics. Revised edition. NY: Oxford University Press.

Goldenberg, P. & Cuoco, A. (1998). What is dynamic geometry? In R.Leher & D. Chazan (Eds.), *Designing learning environments for developing understanding of geometry and space*, pp. 351-367. Mahwah, NJ: Lawrence Erlbaum Associates.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*.

Polya, G. (1945). How to solve it. Princeton: Princeton University Press.

Santos, M. & Díaz Barriga, E. (in press). Posing questions from proposed problems: The role of technology in enhancing mathematical problem solving. *Mathematics Teacher*.

Schoenfeld, A. (1985). *Mathematical problem solving*. New York: Academic Press.

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